Self Assignment: Hypothesis Testing and Confidence Intervals

2024

Introduction

In statistics, **hypothesis testing** and **confidence intervals** are essential concepts used to make inferences about populations from sample data. These methods help in understanding the reliability and significance of the results obtained from data analysis.

1 Hypothesis Testing

1.1 Definition

Hypothesis testing is a statistical method used to make decisions about a population parameter based on sample data. A hypothesis is an assumption about a population parameter, such as the mean or proportion.

There are two types of hypotheses:

- **Null Hypothesis** (H_0): This is the statement we are testing. It represents no effect or no difference.
- **Alternative Hypothesis** (H_a): This represents what we suspect is true instead of the null hypothesis.

1.2 Types of Errors

In hypothesis testing, two types of errors can occur:

- **Type I Error**: Rejecting the null hypothesis when it is actually true. The probability of committing this error is denoted by α, which is called the *significance level*.
- **Type II Error**: Failing to reject the null hypothesis when the alternative hypothesis is true. The probability of committing this error is denoted by β.

1.3 Steps in Hypothesis Testing

The basic steps in hypothesis testing are:

1. **Formulate the Hypotheses**: Set up the null hypothesis (H_0) and the alternative hypothesis (H_a).
2. **Choose the Significance Level** (α): Common choices are 0.05, 0.01, or 0.10, depending on the context.
3. **Calculate the Test Statistic**: Depending on the type of data and the hypothesis, different test statistics such as Z, T, Chi-square, or F-statistics are used.
4. **Determine the Critical Value or p-value**: The critical value is the cutoff point, or you can calculate a p-value, which tells you how likely it is to get a result as extreme as, or more extreme than, the one observed.
5. **Decision Rule**: If the p-value is less than α, reject the null hypothesis (H_0). Otherwise, fail to reject it.

1.4 Common Hypothesis Tests

- **Z-test**: Used when the sample size is large ($n \geq 30$) and the population variance is known.
- **T-test**: Used when the sample size is small ($n < 30$) and the population variance is unknown.
- **Chi-square test**: Used to test for relationships between categorical variables.
- **F-test**: Used to compare two variances.

2 Confidence Intervals

2.1 Definition

A **confidence interval** (CI) is a range of values used to estimate a population parameter with a certain degree of confidence. It provides an interval within which we expect the true population parameter to lie.

The most commonly used confidence levels are 90%, 95%, and 99%. For example, a 95% confidence interval means that we are 95% confident that the true population parameter lies within the interval.

2.2 Formula for Confidence Interval

For a population mean μ, the confidence interval is calculated as:

$$CI = \bar{x} \pm Z \left(\frac{\sigma}{\sqrt{n}} \right)$$

Where:

- $\bar{x}$ is the sample mean,
- Z is the Z-score corresponding to the desired confidence level,
- σ is the population standard deviation, and
- n is the sample size.

2.3 Interpretation

A confidence interval gives an estimated range of values which is likely to include an unknown population parameter. For example, if you compute a 95% confidence interval for the population mean and get the result [10, 15], you can say that you are 95% confident that the true mean lies between 10 and 15.

2.4 Relation to Hypothesis Testing

There is a close relationship between confidence intervals and hypothesis testing. If a confidence interval contains the hypothesized value of the parameter (usually 0), we fail to reject the null hypothesis. If the interval does not contain the hypothesized value, we reject the null hypothesis.

3 Examples

3.1 Example 1: Hypothesis Testing for a Mean

Suppose we are testing whether the mean weight of a type of fruit is 200 grams. We take a random sample of 30 fruits, and the sample mean is 195 grams with a standard deviation of 10 grams. We want to test this at a 5% significance level.

1. H_0: The mean weight is 200 grams ($\mu = 200$).
2. H_a: The mean weight is not 200 grams ($\mu \neq 200$).
3. Significance level $\alpha = 0.05$.
4. Test statistic:
$$Z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}} = \frac{195 - 200}{10/\sqrt{30}} = -2.74$$
5. Using a Z-table, we find the p-value is 0.0062. Since $p < 0.05$, we reject the null hypothesis.

3.2 Example 2: Confidence Interval for a Mean

Using the same sample of 30 fruits with a mean of 195 grams and a standard deviation of 10 grams, let's calculate a 95% confidence interval for the mean weight.

$$CI = \bar{x} \pm Z\left(\frac{\sigma}{\sqrt{n}}\right) = 195 \pm 1.96\left(\frac{10}{\sqrt{30}}\right)$$

$$CI = 195 \pm 3.58$$

$$CI = [191.42, 198.58]$$

We are 95% confident that the true mean weight of the fruits lies between 191.42 grams and 198.58 grams.