MA1101R
AY20/21 sem 2
github.com/jovyntls

- **Ex2.24**
  - (a) if A and B are diagonal matrices of the same size, then $AB = BA$
  - (b) if A is a square matrix, then $(A + A^T)$ is symmetric.
  - (g) if $AA^T = 0$, then $A = 0$.
- **Ex2.61** - if $A = PBP^{-1}$ then $\det(A) = \det(B)$.
- **Ex3.24** - if V and W are subspaces of $\mathbb{R}^n$,
  - $V \cap W$ is a subspace of $\mathbb{R}^n$
  - $V \cup W$ is a subspace of $\mathbb{R}^n \leftrightarrow V \subseteq W$ or $W \subseteq V$
- **Ex3.30** - let $u_1, u_2, \dots, u_k$ be vectors in $\mathbb{R}^n$ and P be a square matrix of order n.
  - if $Pu_1, Pu_2, \dots, Pu_k$ are linearly independent, then $u_1, u_2, \dots, u_k$ are linearly independent
  - if P is invertible and $u_1, u_2, \dots, u_k$ are linearly independent, then $Pu_1, Pu_2, \dots, Pu_k$ are linearly independent
  - if P is NOT invertible and $u_1, u_2, \dots, u_k$ are linearly independent, then $Pu_1, Pu_2, \dots, Pu_k$ are NOT necessarily linearly independent
- **Ex4.10** - the linear relations between columns are not changed by row operations.
- **Ex4.22** - let A be a $m \times n$ matrix and P be a $m \times m$ matrix. if P is invertible, $rank(PA) = rank(A)$
- **Ex4.25** - let A be a $m \times n$ matrix.
  - The nullspace of A is equal to the nullspace of A^TA.
  - nullity(A) = nullity(A^TA)
  - rank(A) = rank(A^TA)
- **Ex5.32** - Let A be an orthogonal matrix. u, v are vectors in $\mathbb{R}^n$.
  - $\|u\| = \|Au\|$
  - $d(u, v) = d(Au, Av)$
  - angle between u and v = angle between Au and Av
- **Ex5.32** - Let A be an orthogonal matrix and $S = \{u_1, u_2, \dots, u_n\}$ be a basis for $\mathbb{R}^n$.
  - $T = \{Au_1, Au_2, \dots, Au_n\}$ is a basis for $\mathbb{R}^n$.
  - if S is orthogonal, T is orthogonal.
  - if S is orthonormal, T is orthonormal.
- **Ex5.34** - A is an orthogonal matrix $\Leftrightarrow$ the columns/rows of A form an orthonormal basis for $\mathbb{R}^n$.
- **Ex6.23** - if A is diagonalisable, A^T is diagonalisable
- **Ex6.26** - if A is symmetric and u, v are 2 eigenvectors of A associated with λ and μ, where $\lambda \neq \mu$, then $u \cdot v = 0$.
- **Ex7.10** - a linear operator T is an isometry if $\|T(u)\| = \|u\|$ for all $u \in \mathbb{R}^n$.
  - (a) $T(u) \cdot T(v) = u \cdot v$ for all $u, v \in \mathbb{R}^n$
  - (b) T is an isometry $\leftrightarrow$ the standard matrix is an orthogonal matrix
  - (c) all isometries on $\mathbb{R}^n$ are of the form
    $T(\begin{pmatrix} x \\ y \end{pmatrix}) = \begin{pmatrix} x\cos\theta + \delta y \sin\theta \\ y\sin\theta - \delta y\cos\theta \end{pmatrix}$ for $\begin{pmatrix} x \\ y \end{pmatrix} \in \mathbb{R}^2$ where $\delta = \pm 1$ and $0 \leq \theta < 2\pi$
- **LAB4** - if AA^T is a diagonal matrix, then the rows of A form an orthogonal set.
- **LAB4** - if AA^T is an identity matrix, then the rows of A form an orthonormal set.
- to show A is invertible: show $\exists B$ s.t. $AB = I$ and $BA = I$