CM1014 Computational Mathematics - Types of Relations

1 Introduction

- One-to-one (Injective)
- Onto (Surjective)
- Bijective
- Reflexive
- Symmetric
- Transitive
- Equivalence Relation
- Partial Order Relation

Each of these relations plays a fundamental role in mathematical structures and functions. We will illustrate them with examples and diagrams.

2 One-to-One (Injective) Relation

A function $f : A \to B$ is **one-to-one** (injective) if different elements in A map to different elements in B, meaning:

$$\forall x_1, x_2 \in A, \quad f(x_1) = f(x_2) \Rightarrow x_1 = x_2.$$

2.1 Example

Consider $f(x) = 2x$, where $x \in \mathbb{R}$. If $f(x_1) = f(x_2)$, then:

$$2x_1 = 2x_2 \Rightarrow x_1 = x_2.$$

Thus, $f(x)$ is injective.

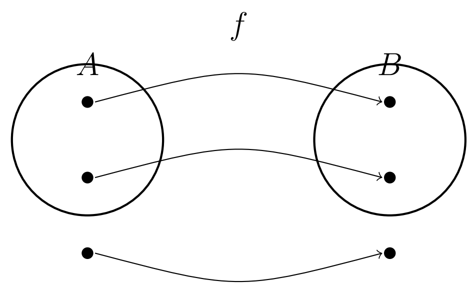

Figure 1: Injective Function: Each element in A maps to a unique element in B.

3 Onto (Surjective) Relation

A function $f : A \to B$ is **onto** (surjective) if every element in B is mapped by at least one element in A, meaning:

$$\forall y \in B, \quad \exists x \in A \text{ such that } f(x) = y.$$

3.1 Example

Consider $f(x) = x^3$. For any $y \in \mathbb{R}$, there exists some x such that $x^3 = y$, so f is surjective.

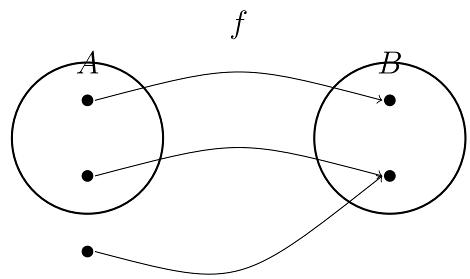

Figure 2: Surjective Function: Every element in B is mapped by at least one element in A.

4 Bijective Relation

A function is **bijective** if it is both injective and surjective, meaning every element in B is uniquely mapped from A.

4.1 Example

The function $f(x) = x + 1$ over $\mathbb{R}$ is bijective since it is both one-to-one and onto.

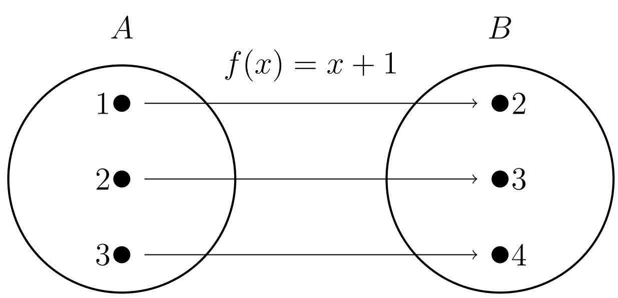

Figure 3: Bijective Function: Each element in A uniquely maps to an element in B and vice versa.

5 Reflexive Relation

A relation R on a set A is **reflexive** if every element is related to itself:

$$\forall x \in A, \quad (x, x) \in R.$$

5.1 Example

The relation $\leq$ on $\mathbb{R}$ is reflexive since $x \leq x$ for all $x \in \mathbb{R}$.

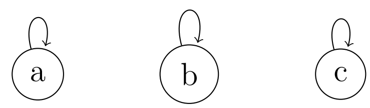

Figure 4: Reflexive Relation: Each element has a loop.

6 Symmetric Relation

A relation R is **symmetric** if:

$$(x, y) \in R \Rightarrow (y, x) \in R.$$

6.1 Example

The equality relation $=$ is symmetric because if $x = y$, then $y = x$.

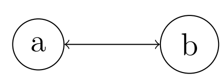

Figure 5: Symmetric Relation: If a is related to b, then b is related to a.

7 Transitive Relation

A relation R is **transitive** if:

$$(x, y) \in R \text{ and } (y, z) \in R \Rightarrow (x, z) \in R.$$

7.1 Example

The relation $\leq$ is transitive since $x \leq y$ and $y \leq z$ implies $x \leq z$.

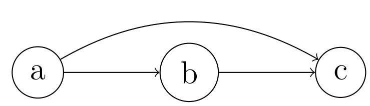

Figure 6: Transitive Relation: If $a \to b$ and $b \to c$, then $a \to c$.

8 Equivalence Relation

A relation is an **equivalence relation** if it is reflexive, symmetric, and transitive.

8.1 Example

The relation R on integers where $x \sim y$ if $x \equiv y \mod 3$ is an equivalence relation.

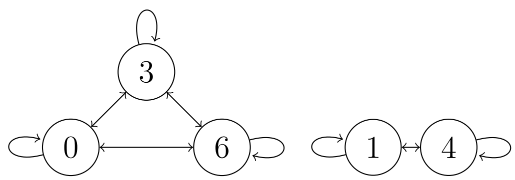

Figure 7: Equivalence Relation: Two equivalence classes modulo 3.

9 Partial Order Relation

A relation is a **partial order** if it is reflexive, antisymmetric, and transitive.

9.1 Example

The subset relation $\subseteq$ is a partial order on the power set of a set.

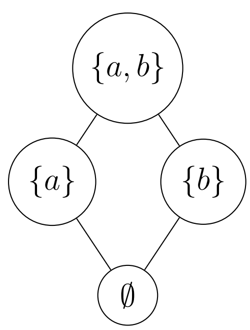

Figure 8: Partial Order Relation: Hasse diagram of the subset relation on $\{\emptyset, \{a\}, \{b\}, \{a, b\}\}$.