MIDTERM ASSESSMENT

UNIVERSITY OF LONDON

CM120

BSc EXAMINATION

COMPUTER SCIENCE

Discrete Mathemaitcs

INSTRUCTIONS TO CANDIDATES:

This assignment consists of five questions. You should answer all five questions.

Full marks will be awarded for complete answers to a total of Five questions. The marks for each part of a question are indicated at the end of the part in [.] brackets.

There are 100 marks available on this paper.

Question 1

(a) Let A and B be two sets with $|A| = 20, |B| = 25$ and $|A \cup B| = 35$. Find $|A \cap B|$. [2]

(b) In a group of 100 people, 73 people can speak English and 43 can speak Spanish.

i. How many can speak both English and Spanish?

ii. How many can speak English only ?

iii. How many can speak Spanish only?

iv. Draw a Venn Diagram to show this information.

[8]

(c) Let A and B be two sets. Determine which of the following statements are true and which are false. Prove each statement that is true and give a counter-example for each statement that is false.

i. $\mathcal{P}(A \cap B) = \mathcal{P}(A) \cap \mathcal{P}(B)$

ii. $\mathcal{P}(A \cup B) = \mathcal{P}(A) \cup \mathcal{P}(B)$

iii. $\mathcal{P}(A) \cup \mathcal{P}(B) \subseteq \mathcal{P}(A \cup B)$

[10]

Question 2

(a) Which of the following are functions? If f is not a function explain why.

i. $f : \mathbb{R} \to \mathbb{R}$ with $f(x) = \frac{1}{1-x^2}$

ii. $f : \mathbb{Z} \to \mathbb{Z}$ with $f(x) = \frac{x}{2}$

iii. $f : \mathbb{R} \to \mathbb{R}$ with $f(x) = \ln(x)$

[3]

(b) Let $f : \mathbb{R} \to \mathbb{R}$ and $g : \mathbb{R} \to \mathbb{R}$ with $f(x) = x + 2$ and $g(x) = -x$. Find $g \circ f, (g \circ f)^{-1}, f^{-1}$ and g^{-1} [4]

(c) Let $f : \mathbb{R}^* \to \mathbb{R}$ with $f(x) = \frac{x+1}{x}$, where $\mathbb{R}^*$ is the set of all real numbers different from 0.

i. Determine whether or not f is a one to one function

ii. Determine whether or not f is an onto function

[4]

(d) Given a function $F : \mathcal{P}(\{a, b, c\}) \to \mathbb{Z}$ is defined by $F(A) = |A|$ for all $A \in \mathcal{P}(\{a, b, c\})$.

i. Is F a one-to-one function? Prove or give a counter-example.

ii. Is F an onto function? Prove or give a counter-example.

[4]

(e) Let $f : A \to B$ and $g : B \to C$ be functions. Prove that if $g \circ f$ is one-to-one then f is also one-to-one. [5]

Question 3

(a) Let p, q, r and s be four propositions. Assuming that p and r are false and that q and s are true, find the truth value of each of the following propositions:

i. $((p \wedge \neg q) \rightarrow (q \wedge r)) \rightarrow (s \vee \neg q)$

ii. $((p \vee q) \wedge (q \vee s)) \rightarrow ((\neg r \vee p) \wedge (q \vee s))$

[4]

(b) Let p, q and r be three propositions defined as follows: p means ' Sofia is happy ' , q means 'Sofia paints a picture ' whereas r means ' Samir is happy '

Express each of the three following compound propositions symbolically by using p, q and r, and appropriate logical symbols.

i. *'If Sofia is happy and paints a picture then Samir isn't happy'*

ii. *'Sofia is happy only if she paints a picture'*

iii. *'Sofia either paints a picture or she is not happy'*

[6]

(c) Give the contrapositive, the converse and the inverse of each of the following statement:

i. $\forall x \in \mathbb{R},$ if $x > 3$ then $x^2 > 9$

ii. $\forall x \in \mathbb{R},$ if $x(x+1) > 0$ then $x > 0$ or $x < -1$

[6]

(d) A tautology is a proposition that is always true. Let p, q and r be three propositions, show that $(p \rightarrow (q \vee r) \Leftrightarrow ((p \wedge \neg q) \rightarrow r)$ is a tautology. [4]

Question 4

(a) Let $P(x, y)$ be a boolean function. Assume that $\exists x \forall y P(x, y)$ is $True$ and that the domain of discourse is nonempty. Which of the following must also be true? If the statement is true, explain your answer; otherwise, give a counter-example.

i. $\forall x \exists y \neg P(x, y)$

ii. $\forall x \forall y P(x, y)$

iii. $\exists x \exists y P(x, y)$

[6]

(b) Given the following argument:

" The football game will be cancelled only If it rains "

"it rained, therefore the football game was cancelled"

Assume p means " it rains" whereas q means "football game cancelled"

i. Translate this argument to a symbolic form.

ii. Construct the truth table.

iii. Determine if this argument is a valid argument or not.

[6]

(c) Say whether or not the following argument is a valid argument. Explain your answer.

(a) Successful candidates for this job must have either a Master's degree or five years of work experience
(b) Johnny has a Master's degree
(c)Johnny got the job

(d) $\therefore$ Johnny does not have five years of work experience [4]

(d) Let $P(x)$ and $Q(x)$ be two predicates and suppose D is the the domain of x. For the statement forms in each pair, determine whether they have the same truth value for every choice of $p(x)$, $Q(x)$ and D, or not.

i. $\forall x \in D, (P(x) \wedge Q(x))$, and $(\forall x \in D, P(x)) \wedge (\forall x \in D, Q(x))$

ii. $\forall x \in D, (P(x) \vee Q(x))$, and $(\forall x \in D, P(x)) \vee (\forall x \in D, Q(x))$

[4]

Question 5

(a) Use the rules of boolean algebra to simplify the following boolean expressions:

i. $\overline{ab}(\overline{a}+b)(\bar{b}+b)$

ii. $\overline{a}(a+b)+(b+aa)(a+\overline{b})$

[6]

(b) Use the duality principle to find out the dual of the following equation:

$$ab+c\overline{d}=(a+c)(a+\overline{d})(b+c)(b+\overline{d})$$

[2]

(c) The lights in a classroom are controlled by two switches: one at the back and one at the front of the room. Moving either switch to the opposite position turns the light off if they were on and on if the were off. Assume the lights have installed so that when both switches are in the down position, the lights are off. Let P and Q be input to switches and R be the output (light on/off), design a circuit to control the witches and give its corresponding truth table. [6]

(d) i. Fill in the following K-map for the Boolean function

$$F(x,y,z)=\overline{x}.\overline{y}.\overline{z}+\overline{x}.\overline{y}.z+x.\overline{y}.\overline{z}+x.\overline{y}.z$$

z \ xy	00	01	11	10
0				
1				

[3]

ii. Use the previous K-map and find a minimisation, as the sum of three terms, of the expression

$$F(x,y,z)=\overline{x}.\overline{y}.\overline{z}+\overline{x}.\overline{y}.z+x.\overline{y}.\overline{z}+x.\overline{y}.z$$

[3]

END OF PAPER

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