Midterm Assessment Coursework

UNIVERSITY OF LONDON

BSc, CertHE and Diploma Examination

COMPUTER SCIENCE

Discrete Mathematics

This paper consists of five questions. You should answer all five questions.

Full marks will be awarded for complete answers to a total of Five questions. The marks for each part of a question are indicated at the end of the part in [.] brackets.

There are 100 marks available on this paper.

Question 1 Set Theory

(a) i. Describe the following set using the listing method:

$$A = \{x : x \in \mathbb{Z} \text{ and } x^2 < 100\}$$

[2]

ii. Rewrite the following set using the set builder method:

$$B = \{2, 5, 8, 11, 14, 17, 20\}$$

[2]

(b) Given two sets A and B, Use the membership table to show that $(A-B) \cup (B-A) = (A \cup B) - (A \cap B)$ [6]

(c) Let $\star$ be a new operator on sets is defined as follows: Given two sets A and B, $A \star B = \overline{A \cap B}$.

i. Draw a Venn Diagram for $A \star B$ [2]

ii. By using the laws of the algebra of sets show that the following is true:

- $(A \star A) \star (B \star B) = A \cup B$ [2]
- $(A \star B) \star (A \star B) = A \cap B$ [2]

(d) Given three sets A, B and C, use the laws of the algebra on sets to show that $(A \cup B \cup C) \cap (A \cup \overline{B} \cup C) \cap \overline{(A \cup C)} = \emptyset$ [4]

Question 2 Functions

(a) Let $f(x) = x$ mod 3, where $f(x)$ is the remainder when x is divided by 3, and $f : \mathbb{N} \to \{0, 1, 2\}$.

i. Find $f(7)$ and $f(12)$. [1]

ii. What is the set of pre-images of 2? [1]

iii. Say whether or not $f(x)$ is injective(one-to-one), justifying your answer. [2]

iv. Say whether or not $f(x)$ is surjective(onto), justifying your answer. [2]

(b) Let $B = \{x : x \in \mathbb{R}$ and $x \neq 1\}$ and $g : B \to B$ is defined by $g(x) = \frac{x}{x-1}$.

i. Show that the function g is a bijection. [3]

ii. Find g^{-1}. [2]

(c) Let $f : \mathbb{R} \to]1, +\infty[$ with $f(x) = 2^x + 1$.

i. Find the inverse function f^{-1}. [2]

ii. Plot the curves of both function, f and f^{-1} in the same graph. [2]

iii. What can you say about these two curves? [1]

(d) Let A and B be two finite sets with $|A| = |B|$ and f be a function from A to B. show that if f is an injective (one-to-one) function then it is also a surjective function. [4]

Question 3 Propositional Logic

(a) Let $n \in \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$ and let p and q be two propositions concerning the integer n.

$p : n$ is even and $q : n \geq 5$

Draw up the appropriate truth table and find the truth set for each of the propositions $p \oplus \neg q$ and $\neg(p \to q)$. [6]

(b) Let p and q be two propositions defined as follows: p means 'The home football team wins' whereas q means 'it's raining'

i. Express each of the four following compound propositions symbolically by using $p,\ q$ and appropriate logical symbols.

'The home football team wins whenever it's raining'

'neither the home football team wins nor it's raining'

' Either the home football wins or it's raining, but not both'

'The home football team wins only if it's raining'

[4]

ii. What are the contrapositive, the converse and the inverse of the implication
'The home football team wins only if it's raining' [2]

(c) Consider the following four propositions:

s means '"Samir comes to the party"

c means '"Callum comes to the party"

r means '"Ruby comes to the party"

j means '"Jay comes to the party".

Express each of the three following compound propositions symbolically by using s, c, r, j and appropriate logical symbols.

i. "If Samir comes to the party and Callum does not come to the party, then Jay will come to the party .

ii. "If Jay comes to the party, then, if Callum doesnt come then both Samir and Ruby come.

[4]

(d) A tautology is a proposition that is always true. Let p, q and r be three propositions, show that $(p \vee q) \wedge (\neg p \vee r) \rightarrow (q \vee r)$ is a tautology. [4]

Question 4 Predicate logic

(a) Let A be the set of Discrete Mathematics students, B the set of students of Boolean Algebra and S the set of all students. Use rules of inference with quantifiers to formalise the three following statements:

P : If someone is a student of Discrete Mathematics, then, they must study Boolean Algebra".

Q : If there exists at least one student of Discrete Mathematics, then, all students of Boolean Algebra study Discrete Mathematics.

R : If all students of Boolean Algebra study Discrete Mathematics then nobody studies Discrete Mathematics". [6]

(b) Prove that the following hypotheses: $H1, H2, H3$ and $H4$ implies the proposition: $\overline{T}$, where :

$H1$: $P \wedge Q$,
$H2$: $P \rightarrow \overline{Q \wedge S}$,
$H3$: $R \rightarrow S$,
$H4$: $T \wedge P \rightarrow R$

[9]

(c) The following proposition R is wrong :

R : if $\exists x P(x) \wedge \exists x Q(x)$ is true then $\exists x(P(x) \wedge Q(x))$ is true

The following proof of R, using inference rules with quantifiers is supposed to prove the proposition R, but it is also wrong.
Identify the erroneous step or steps in this reasoning and explain why it is wrong :

H: $\exists x P(x) \wedge \exists x Q(x)$,
Step1 : By *Simplification* from H, we get: $\exists x P(x)$
Step2 : By *Existential instantiation* from **Step1**, we get: $P(c)$
Step3 : By *Simplification* from H, we get : $\exists x Q(x)$
Step4 : By *Existential instantiation* from **Step3**, we get : $Q(c)$
Step5 : By *Conjunction* from **Step3** and **Step4**, we get : $P(c) \wedge Q(c)$
Step6 : By *Existential generalization* from **Step5**, we get : $\exists x(P(x) \wedge Q(x))$ [5]

Question 5 Boolean Algebra

(a) Which theorems, explained in the lecture, represents each one of the the following Boolean statements :
$P : (a + b) + c = a + (b + c)$
$Q : a + a = a$
$R : a + ab = a$ [6]

(b) Build the truth table of the Boolean function $F(x, y, z)$ that equals 1 if and only if $x.y.z = 0$ [2]

(c) Use the previous truth table to find the sum-of-products expansions of the Boolean function $F(x, y, z)$. [2]

(d) i. Give two reasons why circuit minimisation is beneficial in circuit design. [2]

ii. To produce a sum-of-product algebraic simplification , we usually use theorems (laws). Give three examples of theorems we can use. [3]

iii. Fill in the following K-map for the Boolean function

$$F(x, y, z) = x.y.z + x.\overline{y}.z + x.\overline{y}.\overline{z} + \overline{x}.y.z + \overline{x}.y.\overline{z} + \overline{x}.\overline{y}.\overline{z}$$

	$\overline{y}.\overline{z}$	$\overline{y}.z$	$y.z$	$y.\overline{z}$
$\overline{x}$				
x				

[2]

iv. Use the previous K-map and find a minimisation, as the sum of three terms, of the expression $F(x, y, z) = x.y.z + x.\overline{y}.z + x.\overline{y}.\overline{z} + \overline{x}.y.z + \overline{x}.y.\overline{z} + \overline{x}.\overline{y}.\overline{z}$ [3]

END OF PAPER