UNIVERSITY OF LONDON

CM1020

BSc EXAMINATION

COMPUTER SCIENCE

Discrete Mathematics

Release date: Monday 7 March 2022 at 12:00 midday Greenwich Mean Time

Submission date: Tuesday 8 March 2022 by 12:00 midday Greenwich Mean Time

Time allowed: 24 hours to submit

INSTRUCTIONS TO CANDIDATES:

Section A of this assessment consists of a set of **TWENTY** Multiple Choice Questions (MCQs) which you will take separately from this paper. You should attempt to answer **ALL** the questions in Section A. The maximum mark for Section A is 40.

Section A will be completed online on the VLE. You may choose to access the MCQs at any time following the release of the paper, but once you have accessed the MCQs you must submit your answers before the deadline or within **4 hours** of starting whichever occurs first.

Section B of this assessment is an online assessment to be completed within the same 24-hour window as Section A. We anticipate that approximately **1 hour** is sufficient for you to answer Section B. Candidates must answer **TWO** out of the THREE questions in Section B. The maximum mark for Section B is 60.

You may use any calculator for any appropriate calculations, but you may not use computer software to obtain solutions. Credit will only be given if all workings are shown.

You should complete Section B of this paper and submit your answers as **one document**, if possible, in Microsoft Word or a PDF to the appropriate area on the VLE. We advise you to upload as few documents as possible. Each file uploaded must be accompanied by a coversheet containing your **candidate number**. In addition, your answers must have your candidate number written clearly at the top of the page before you upload your work. Do not write your name anywhere in your answers.

UL22/0095

SECTION A

Candidates should answer the **TWENTY** Multiple Choice Questions (MCQs) quiz, **Question 1** in Section A on the VLE.

SECTION B

Candidates should answer any **TWO** questions from Section B.

Question 2

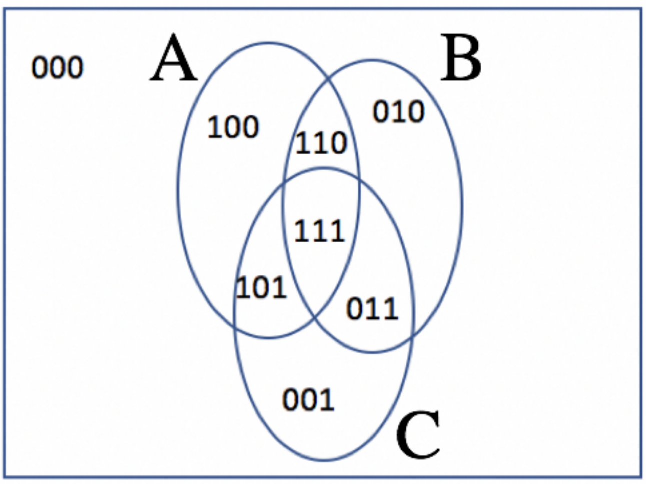
(a) Let U be the set of the total number of students at a school, A be the set of students who play football and B be the set of students who play basketball. There are 240 students in the school:

- 120 students play football
- 40 students play basketball
- 90 students play football but not basketball.

i. Draw a Venn diagram that represents this information [2]

ii. Find the cardinality of the set $A' \cap B'$ [2]

(b) Consider the following Venn diagram representing three sets A, B and C intersecting in the most general way. Three binary digits are used to refer to each one of the eight regions in this diagram. In terms of A, B and C, write the set representing the area comprising the regions 011, 100 and 101.



[3]

(c) Let p, q and r be three propositions for which p and q are true, and r is false. Determine the truth value for each of the following: [3]

i. $p \to (r \to q)$

ii. $(p \oplus r) \to \neg q$

iii. $p \wedge (r \to q)$

(d) Let $P(x)$ be the statement " $x^2 > 1$ " and $Q(x)$ be the statement "$x+1 < 4$". The universe of discourse consists of all real numbers. What are the truth values for the following:

i. $\forall x(P(x) \to Q(x))$

ii. $\exists x(P(x) \to Q(x))$

iii. $\forall x(P(x) \wedge Q(x))$

iv. $\exists x(P(x) \wedge \neg Q(x))$

[8]

(e) Decide if the following arguments are valid or invalid. State the Rule of Inference or fallacy used.

i. If it snows, then school is closed
School is open
$\therefore$ it is not snowing.

ii. If the movie is long, I will fall asleep
I do fall asleep.
$\therefore$ the movie was long.

[4]

(f) Let A, B and C be three sets. Prove by contradiction that if $A \cap B \subseteq C$ and $x \in B$, then $x \notin A - C$. [4]

(g) Suppose there are 5 questions in an exam paper, find the number of ways in which a student can attempt one or more questions. [4]

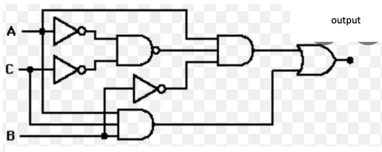
Question 3

(a) Minimise the following logic function using the Karnaugh maps method:

$$f(x, y, z) = x'y'z' + x'y + xyz' + xz$$

[6]

(b) Given the following logical circuit with three inputs A, B and C:



i. Use the boolean algebra notation and write down the boolean expression of the output of this circuit. [4]

ii. Simplify the logical expression in (i). Explain your answer. [5]

(c) Determine with justification whether each of the following functions $\mathbb{R} \to \mathbb{R}$ is one-to-one, onto or bijective.

i. $f_1(x) = 2x + 5$

ii. $f_2(x) = x^2 + 2x + 1$

iii. $f_3(x) = \exp^{-x}$

[9]

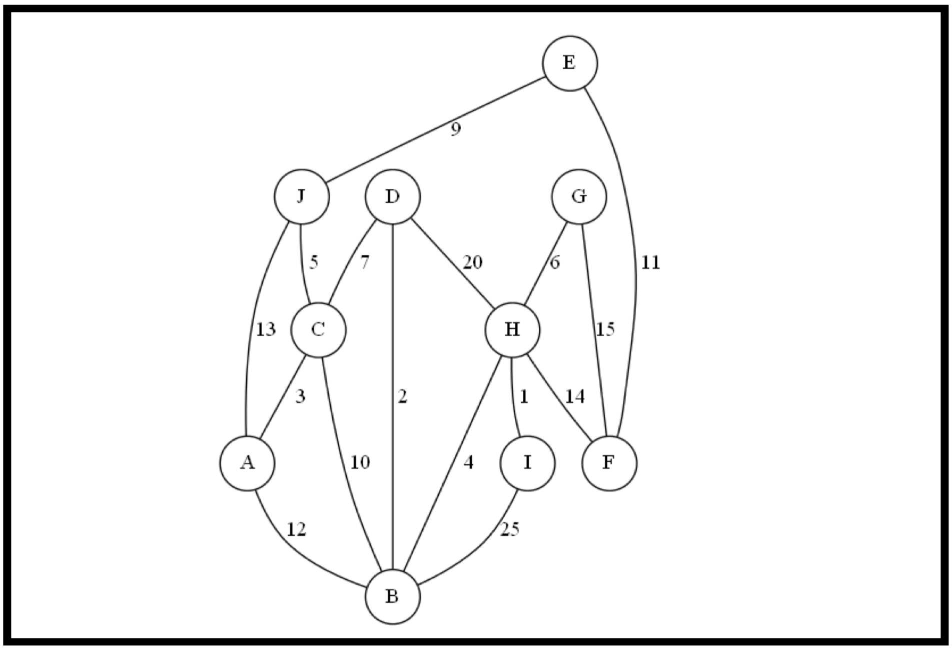
(d) Prove by induction that $5^{\ n} - 1$ is divisible by 5 for all $n \in \mathbb{Z}^{\ +}$. [6]

Question *4*

(a) Explain the difference between a Hamiltonian cycle and an Euler cycle. [3]

(b) Find the maximum number of comparisons to be made to find any record in a binary search tree which holds 5000 records. [3]

(c) Use Prim's algorithm starting at node A to compute the Minimum Spanning Tree (MST) of the following weighted graph. In particular, write down the edges of the MST in the order in which Prim's algorithm adds them to the MST. Use the format *(node 1, node 2)* to denote an edge. Find the cost of this MST.



[6]

(d) Give an example of an equivalence relation on the set $\{1, 2, 3\}$ with exactly two equivalence classes. [2]

(e) Given S is the set of integers $\{2, 3, 4, 6, 7, 9\}$. Let $\mathcal{R}$ be a relation defined on S by the following condition such that,
for all $x, y \in S$, xRy if $3|(x + y)$ which means 3 divides $(x - y)$.

i. Draw the digraph of $\mathcal{R}$. [2]

ii. Say with reason whether or not $\mathcal{R}$ is

- reflexive;
- symmetric;
- anti-symmetric;
- transitive.

In the cases where the given property does not hold, provide a counter example to justify this. [6]

iii. is $\mathcal{R}$ a partial order? Explain your answer. [1]

iv. is $\mathcal{R}$ an equivalence relation? [1]

(f) Let$f : A \to B$ and $g : B \to C$ be functions. Prove that if $g\ o\ f$ is one-to-one, then f is one-to-one. [6]

END OF PAPER

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