PROBLEM SHEET 6

6.1 Reduce to standard form

$$(a)\ \ \frac{3+i}{4-i}, \quad \text{and} \quad (b)\ \ (1+i)^5.$$

6.2 Prove

$$(a)\ \ |z_1 z_2| = |z_1|\,|z_2|, \quad \text{and} \quad (b)\ \ \left|\frac{z_1}{z_2}\right| = \frac{|z_1|}{|z_2|} \ \text{ when } z_2 \neq 0.$$

6.3 Given that $e^{i\theta} = \cos\,\theta + i\,\sin\,\theta$, prove that

$$\cos(A+B) = \cos A \cos B - \sin A \sin B.$$

6.4 Let $z = 1+i$. Find the following complex numbers in standard form and plot their corresponding points in the Argand diagram:-

$$(a)\ \ \bar{z}^2, \quad \text{and} \ \ (b)\ \ \frac{z}{\bar{z}}.$$

6.5 Find the modulus and principal arguments of (a) $-2+2i$, (b) $3+4i$.

6.6 Find all the complex roots of
(a) $\cosh z = 1$;
(b) $\sinh\,z = 1$;
(c) $e^z = -1$;
(d) $\cos z = \sqrt{2}$.

6.7 Show that the mapping

$$w = z + \frac{c}{z},$$

where $z = x+iy$, $w = u+iv$ and c is a real number, maps the circle $|z| = 1$ in the z plane into an ellipse in the w plane and find its equation.

6.8 Show that

$$\cos^6\theta = \frac{1}{32}(\cos 6\theta + 6\cos 4\theta + 15\cos 2\theta + 10).$$