PROBLEM SHEET 5

5.1 Verify the following Taylor expansions (taking the ranges of validity for granted).

(a)

$$e^x = 1 + \frac{1}{1!}x + \frac{1}{2!}x^2 + \frac{1}{3!}x^3 + ... + \frac{1}{n!}x^n + ...\text{valid for any } x.$$

(b)

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - ... + \frac{(-1)^n x^{2n+1}}{(2n+1)!} + ...\text{valid for any } x.$$

(c)

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - ... + \frac{(-1)^n x^{2n}}{(2n)!} + ...\text{valid for any } x.$$

(d) Let α be a constant.

$$(1+x)^\alpha = 1 + \alpha x + \frac{\alpha(\alpha-1)}{2!}x^2 + \frac{\alpha(\alpha-1)(\alpha-2)}{3!}x^3 + ...\text{valid for } -1 < x < 1$$

(e)

$$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - ... + \frac{(-1)^{n-1}x^n}{n} + ...\text{valid for } -1 < x \leq 1.$$

5.2 Obtain a four-term Taylor polynomial approximation valid near $x = 0$ for each of the following.

$$(a)\ \ (1+x)^{1/2}, \qquad (b)\ \ \sin(2x), \qquad (c)\ \ \ln(1+3x).$$