PROBLEM SHEET 2

2.1 Given that

$$\sinh x = \frac{1}{2}[e^x - e^{-x}]$$

show that

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \cosh x.$$

2.2 Given that

$$\cosh x = \frac{1}{2}[e^x + e^{-x}],$$

show that

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \sinh x.$$

2.3 Let n be a positive integer. Show that

$$\frac{\mathrm{d}^n(x^n)}{\mathrm{d}x^n} = n!$$

2.4 If $y = \ln x$, show that

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{1}{x}; \qquad \frac{\mathrm{d}^2y}{\mathrm{d}x^2} = \frac{-1}{x^2}; \qquad \frac{\mathrm{d}^{100}y}{\mathrm{d}x^{100}} = \frac{-99!}{x^{100}}.$$

2.5 Find the equation of the tangent to the curve $y = x^2$ at $(1, 1)$.

2.6 Find the slope of the curve $y = 4x + e^x$ at $(0, 1)$.

2.7 Find the angle of inclination of the tangent to the curve $y = x^2 + x + 1$ at the point $(0, 1)$.

2.8 The displacement $y(t)$ metres of a body at time t seconds ($t \geq 0$) is given by $y(t) = t - \sin t$. At what times is the body at rest?

2.9 A particle has displacement $y(t)$ metres at time t seconds given by $y(t) = 3t^3 + 4t + 1$. Find its acceleration at time $t = 4$ seconds.

2.10 If

$$y = \sum_{n=0}^{N} a_n x^n$$

show that

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \sum_{n=1}^{N} n a_n x^{n-1}.$$