Problems for Calculus 2

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Easy Problems

1. Differentiate $f(x) = 4x^3 - 5x + 7$.

$$f'(x) = 12x^2 - 5$$

2. Compute $\lim_{x \to 1} \frac{x^2-1}{x-1}$.

$$\lim_{x \to 1} \frac{x^2 - 1}{x - 1} = 2$$

3. Find the integral $\int (2x - 1)\, dx$.

$$\int (2x - 1)\, dx = x^2 - x + C$$

4. Compute the limit $\lim_{x \to 0} \frac{1-\cos(x)}{x^2}$.

$$\lim_{x \to 0} \frac{1 - \cos(x)}{x^2} = \frac{1}{2}$$

5. Find the derivative of $f(x) = \ln(x^2 + 1)$.

$$f'(x) = \frac{2x}{x^2 + 1}$$

Medium Problems

1. Differentiate $f(x) = e^x \sin(x)$ using the product rule.

$$f'(x) = e^x \sin(x) + e^x \cos(x)$$

2. Evaluate the definite integral $\int_0^2 (x^2 + 3x)\, dx$.

$$\int_0^2 (x^2 + 3x)\, dx = \frac{28}{3}$$

3. Solve $\lim_{x \to \infty} \frac{3x^2+2x}{x^2-4}$.

$$\lim_{x \to \infty} \frac{3x^2 + 2x}{x^2 - 4} = 3$$

4. Find the inflection points of $f(x) = x^3 - 6x^2 + 9x$.

$$f''(x) = 6x - 12, inflection point at x = 2$$

5. Use the chain rule to differentiate $f(x) = \cos(3x)$.

$$f'(x) = -3 \sin(3x)$$

Hard Problems

1. Solve the integral $\int_0^\pi \sin(x) e^{\cos(x)}\, dx$.

$$\int_0^\pi \sin(x) e^{\cos(x)}\, dx = e - 1$$

2. Find the maximum and minimum of $f(x) = x^4 - 4x^3 + 6x^2$ on the interval $[0, 3]$.

$$Max at x = 3, Min at x = 0$$

3. Compute the Maclaurin series for $f(x) = \cos(x)$.

$$f(x) = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \dots$$

4. Solve the differential equation $\frac{dy}{dx} = y - 2x$.

$$y = Ce^x + 2x + 1$$

5. Compute the integral $\int_1^2 \frac{\ln(x)}{x^2}\, dx$.

$$\int_1^2 \frac{\ln(x)}{x^2}\, dx = -\frac{\ln(2)}{2} - \frac{1}{2}$$