Problems for Calculus

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Easy Problems

1. Find the derivative of $f(x) = x^2 + 3x + 2$.

   $$f'(x) = 2x + 3$$

2. Compute the limit $\lim_{x \to 2} \frac{x^2 - 4}{x - 2}$.

   $$\lim_{x \to 2} \frac{x^2 - 4}{x - 2} = 4$$

3. Find the integral $\int 3x^2 \, dx$.

   $$\int 3x^2 \, dx = x^3 + C$$

4. Compute the limit $\lim_{x \to 0} \frac{\sin(x)}{x}$.

   $$\lim_{x \to 0} \frac{\sin(x)}{x} = 1$$

5. Find the critical points of $f(x) = x^3 - 3x^2 + 2x$.

   $$f'(x) = 3x^2 - 6x + 2 = 0, critical points at x = 1 \pm \frac{1}{\sqrt{3}}$$

Medium Problems

1. Apply the product rule to differentiate $f(x) = x^2 \sin(x)$.

$$f'(x) = 2x \sin(x) + x^2 \cos(x)$$

2. Compute the definite integral $\int_0^1 (3x^2 + 2x + 1)\, dx$.

$$\int_0^1 (3x^2 + 2x + 1)\, dx = 2$$

3. Solve the limit $\lim_{x \to \infty} \frac{2x^3 - x}{x^3 + 4x^2}$.

$$\lim_{x \to \infty} \frac{2x^3 - x}{x^3 + 4x^2} = 2$$

4. Find the points of inflection for $f(x) = x^4 - 4x^3 + 6x^2$.

$$f''(x) = 12x^2 - 24x, inflection points at x = 0, 2$$

5. Use the chain rule to differentiate $f(x) = \sin(2x)$.

$$f'(x) = 2 \cos(2x)$$

Hard Problems

1. Solve the integral $\int_0^1 e^x \sin(x)\, dx$.

$$\int_0^1 e^x \sin(x)\, dx = \frac{e - 1}{2}$$

2. Find the maximum and minimum values of $f(x) = x^3 - 3x^2 + 2x$ on the interval $[0, 3]$.

$$Max at x = 3, Min at x = 2$$

3. Compute the Taylor series expansion for $f(x) = e^x$ around $x = 0$.

$$f(x) = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots$$

4. Find the solution to the differential equation $\frac{dy}{dx} = y + x$.

$$y = Ce^x - x - 1$$

5. Compute the integral $\int_0^1 x^2 \ln(x)\, dx$.

$$\int_0^1 x^2 \ln(x)\, dx = -\frac{1}{9}$$