CS4231
AY22/23 SEM 2
github/jovyntls

01. MUTUAL EXCLUSION

properties of a mutex algorithm

- **mutual exclusion** → no more than one process in the critical section
- **progress** → if one or more process wants to enter and if no one is in the critical section, then one of them can eventually enter the critical section
- **no starvation** → if a process wants to enter, it eventually can always enter
  - no starvation implies progress!
- ! if a process is in the CS, we always assume that it will eventually exit the CS
  - finite number of instructions in the CS

Peterson's Algorithm

Shared bool wantCS[0] = false, bool wantCS[1] = false, int turn = 0;

Process 0	*Process 1*

RequestCS(0) {
wantCS[0] = true;
turn = 1;
while (wantCS[1] == true && turn == 1) {};
}
	RequestCS(1) {
	wantCS[1] = true;
	turn = 0;
	while (wantCS[0] == true && turn == 0) {};
	}

ReleaseCS(0) {
wantCS[0] = false;
}
	ReleaseCS(1) {
	wantCS[1] = false;
	}

proof

- **mutual exclusion**: proof by contradiction
  - case 1 - turn == 0 when P0 and P1 are both in CS
    - then P0 executed turn=1 before P1 executed turn=0
    - hence wantCS[0]==false as seen by P1
    - but wantCS[0] set to true by P0
  - case 2 - turn ==1. symmetric
- **progress**: proof by contradiction
  - suppose both want to enter but none can enter ⇒ wantCS must be true for both
  - case 1: turn==0. then P0 can enter
  - case 2: symmetric
- **no starvation**: proof by contradiction
  - case 1: P0 is waiting, then wantCS[1]==true and turn=1
    - P1 in critical region - eventually it exits and sets wantCS[1] to false
    - (what if P1 wants to enter again immediately? then P1 will wait first because wantCS[0]==true and it has set turn==0)
  - case 2: P1 is waiting. symmetric

Lamport's Bakery Algorithm

- for n processes
  - get a number first (weak guarantee)
  - get served when all lower numbers have been served (sufficient for mutex)
- 2 shared arrays of n elements
  - `boolean choosing[i] = false` ⇒ is process i trying to get a number
  - `int number[i] = 0` ⇒ the number gotten by process i
    - number[i] > 0: process wants to enter CS and that is the queue number

```
ReleaseCS(int myid) {
    number[myid] = 0;
}

// a utility function
boolean Smaller(int number1, int id1, int number2, int id2) {
    if (number1 < number 2) return true;
    if (number1 == number2) {
        if (id1 < id2) return true; else return false;
    }
    if (number 1 > number2) return false;
}
```

```
RequestCS(int myid) {
    // get a number
    choosing[myid] = true;
    for (int j = 0; j < n; j++)
        if (number[j] > number[myid]) number[myid] = number[j];
    number[myid]++;
    choosing[myid] = false;

    // wait for people ahead of me
    for (int j = 0; j < n; j++) {
        while (choosing[j] == true);
        while (number[j] != 0 &&
                Smaller(number[j], j, number[myid], myid));
    }
}
```

Hardware Solutions

- **disable interrupts** - to prevent context switch
  - do not allow context switch in the critical section
- special machine-level instructions: **TestAndSet** executed atomically
  - × when you design CPU, you want all instructions to roughly be the same complexity so that your pipelines don't have bubbles in it
  - × degrades performance

```
boolean TestAndSet(Boolean openDoor, boolean newValue) {
    boolean tmp = openDoor.getValue();     // Executed
    openDoor.setValue(newValue);           // atomically
    return tmp;
}
```

```
shared Boolean variable openDoor initialized to true;
RequestCS(process_id) {
    while (TestAndSet(openDoor, false) == false) {};
}
ReleaseCS(process_id) { openDoor.setValue(true); }
```

Proof: Lamport's Bakery Algorithm

```
choosing[myid] = true;
for (int j = 0; j < n; j ++)
    if (number[ j ] > number[myid])
        number[myid] = number[ j ];
number[myid]++;
choosing[myid] = false;

for (int j = 0; j < n; j ++) {
    while (choosing[ j ] == true);
    while (number[ j ] != 0 &&
            Smaller(number[ j ], j,
            number[myid], myid));
}
```

- **Progress**: Proof by contradiction. Consider any set of processes that wants to enter the CS but no one can make progress. Each process is guaranteed to get a queue #. Let process i be the one with the smallest queue number. Consider where process i can be blocked:
  - Case 1: Process j will eventually set choosing[j] to false. Process j will then block (otherwise there is progress already!)
  - Case 2: Impossible since process i has the smallest queue number

Mutual exclusion: Suppose i and k both in critical section.

At T1, process i is here

```
choosing[myid] = true;
for (int j = 0; j < n; j ++)
    if (number[ j ] > number[myid])
        number[myid] = number[ j ];
number[myid]++;
choosing[myid] = false;
```

```
for (int j = 0; j < n; j ++) {
    while (choosing[ j ] == true);
    while (number[ j ] != 0 &&
            Smaller(number[ j ], j,
            number[myid], myid));
}
```

At T1, process k is here

- W.l.o.g, assume Smaller(number[i], i, number[k], k) after they are in the critical sec
- Process k must see number[i] == 0 at that time T1: We want to know where process i is at time T1.
  - Case 1: Process i has not executed "if (number[k] > number[i])".Then eventually number[i] > number[k]. Impossible.
  - Case 2: Has executed "if ()"
    - Subcase 2.1: Process i has executed "number[myid]++;"
    - -- impossible since number[i] == 0
    - Subcase 2.2: Has not executed "number[myid]++;" -- This is the only possible case.

Case 2: At T2, process i is here

At T1, process i is here

```
choosing[myid] = true;
for (int j = 0; j < n; j ++)
    if (number[ j ] > number[myid])
        number[myid] = number[ j ];
number[myid]++;
choosing[myid] = false;
```

At T2, process k is here

```
for (int j = 0; j < n; j ++) {
    while (choosing[ j ] == true);
    while (number[ j ] != 0 &&
            Smaller(number[ j ], j,
            number[myid], myid));
}
```

At T1, process k is here

- Now continue and consider the time T2 when process k invoked "while (choosing[i] = true);" and passed that statement.
- We want to see where process i is at T2. Since choosing[i] = false, process i must either have finished choosing its queue number or have not started choosing:
  - Case 1: process i has finished choosing and has executed choosing[i] = false; Impossible since T2 < T1.
  - Case 2: process i has not started choosing and has executed choosing[i] = true. But then number[i] will be larger than number[k]. Contradiction.

02. SYNCHRONISATION PRIMITIVES

- solves **busy wait problem** (wastes CPU cycles)
- synchronisation primitives: OS-level APIs that the program may call

Semaphores

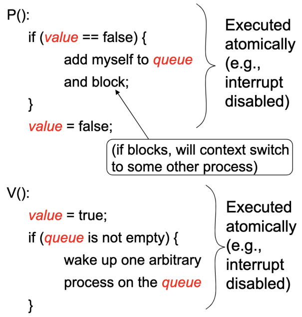
2 variables for each semaphore
- `boolean value := true`
- queue (of blocked processes) `:= empty`

2 APIs, executed atomically

`P()` - wait
- if value == false, add self to queue and block
- can context switch to some other process

`V()` - signal
- set `value = true`
- wake up one arbitrary process in queue

semaphore for mutex:
- `RequestCS() { P(); }`
- `ReleaseCS() { V(); }`

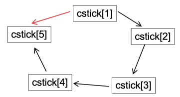


dining philosophers

- one semaphore for each chopstick
- waits-for graph has a cycle $\Rightarrow$ **deadlock**
  - avoid cycles in WFG / have a total ordering

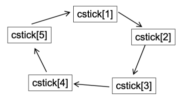
philosopher 1	*philosopher 2*	*philosopher 3*	*philosopher 4*	*philosopher 5*
cstick[1].P();	cstick[2].P();	cstick[3].P();	cstick[4].P();	cstick[1].P();
cstick[2].P();	cstick[3].P();	cstick[4].P();	cstick[5].P();	cstick[5].P();

- Avoid cycles (or have a total ordering of the chopsticks)



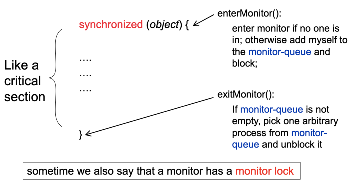
philosopher 1	*philosopher 2*	*philosopher 3*	*philosopher 4*	*philosopher 5*
cstick[1].P();	cstick[2].P();	cstick[3].P();	cstick[4].P();	cstick[5].P();
cstick[2].P();	cstick[3].P();	cstick[4].P();	cstick[5].P();	cstick[1].P();

- It is possible for all processes to block
  - Deadlock !

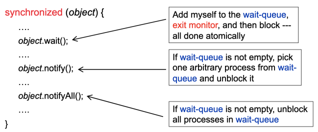


Monitor

- ✓ higher-level/easier to use than semaphore
- every object in Java is a monitor



- Each monitor has two queues of blocked processes
  - monitor-queue and wait-queue
- Three special methods for using when inside a monitor



Hoare-style

- `notify()` immediately switches from caller to a waiting thread
- doesn't use `notifyAll()`

First kind: Hoare-style Monitor

Process 0	*Process 1*
synchronized (object) { if (x != 1) object.wait();	

	synchronized (object) {
	x=1;
	object.notify();

assert(x == 1); // x must be 1
x = 2;
}	

	// x may no longer be 1 here }

process 0 takes over the execution

Java-style

- `notify()` places a waiter on the ready thread but signaller continues inside the monitor

Second kind: Java-style Monitor

Process 0	*Process 1*
synchronized (object) { if (x != 1) object.wait();	

	synchronized (object) {
	x=1;
	object.notify();

	assert(x == 1); // x must be 1
	x=2;
	}

// needs to acquire monitor lock
// x may not be 1 here
}	

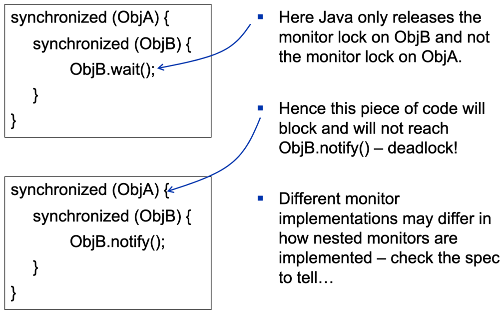
process 1 continues execution

- P0 must acquire monitor lock
  - use `(while x!=1)` to ensure `x==1`

nested monitor

- `wait()` only releases the *immediate* monitor lock

Nested Monitor in Java



producer-consumer problem

- circular buffer of size n
- single producer, single consumer
  - producer places item to the end of the buffer if buffer is not full
  - consumer removes item from the head of the buffer if not empty

object sharedBuffer;

```
void produce() {
  synchronized (sharedBuffer) {
    if (sharedBuffer is full)
      sharedBuffer.wait();
    add an item to sharedBuffer;
    if (sharedBuffer *was* empty)
      sharedBuffer.notify();
  }
}
```

```
void consume() {
  synchronized (sharedBuffer) {
    if (sharedBuffer is empty)
      sharedBuffer.wait();
    remove item from sharedBuffer;
    if (sharedBuffer *was* full)
      sharedBuffer.notify();
  }
}
```

reader-writer problem

- multiple readers and writers accessing a file
  - writer must have exclusive access
  - readers may simultaneously access the file

int numReader, numWriter; Object object;

```
void writeFile() {
  synchronized (object) {
    while (numReader > 0 ||
           numWriter > 0)
      object.wait();
    numWriter = 1;
  }
  // write to file;
  synchronized (object) {
    numWriter = 0;
    object.notifyAll();
  }
}
```

```
void readFile() {
  synchronized (object) {
    while (numWriter > 0)
      object.wait();
    numReader++;
  }
  // read from file;
  synchronized (object) {
    numReader--;
    object.notify();
  }
}
```

you can prove that it must be a writer who is notified

reader-writer (without starvation)

maintain an explicit queue

```
Vector queue; // shared among all threads
int numReader; int numWriter;
// code for writer entry
Writer w = new Writer(myname);
synchronized (queue) {
    if (numReader > 0 || numWriter > 0) {
        w.okToGo = false;
        queue.add(w);
    } else {
        w.okToGo = true;
        numWriter++;
    }
}
synchronized (w) { if (!w.okToGo) w.wait(); }
// code for writer exit
synchronized (queue) {
    numWriter--;
    if (queue is not empty) {
        remove a single writer or a batch of readers from queue
        for each request removed do {
            numberWriter++ or numberReader++;
            synchronized (request) {
                request.okToGo = true;
                request.notify();
            }
        }
    }
}
// code for reader entry
Reader r = new Reader(myname);
synchronized (queue) {
    if ((numWriter > 0) || !queue.isEmpty()) {
        r.okToGo = false;
        queue.add(r);
    } else {
        r.okToGo = true;
        numReader++;
    }
}
synchroized (r) { if (!r.okToGo) r.wait(); }
// code for reader exit
synchronized (queue) {
    numReader--;
    if (numReader > 0) exit(); // I am not the last reader
    if (queue is not empty) {
        remove a single writer or a batch of readers from queue
        for each request removed do {
            numWriter++ or numReader++;
            synchronized (request) {
                request.okToGo = true;
                request.notify();
            }
        }
    }
}
```

barber-shop problem

Customer

```
Vector customQueue; // shared data among all threads

synchronized (numberChair) {
    if (numberChair > 0) numberChair--;
    else return; // leave // this releases monitor lock
}

// middle part (see next slide)

synchronized (numChair) {
    numberChair++;
}
```

Customer – Middle part

```
// middle part
Customer myself = new Customer(myname);
myself.done = false;
synchronized (customQueue) {
    customQueue.add(myself); // can also simulate chair here
    customQueue.notify();
}

synchronized (myself) {
    if (!myself.done) myself.wait(); // no need to use while
}
```

Barber

```
while (true) {
    Customer current;
    synchronized (customerQueue) {
        if (customerQueue.isEmpty()) customerQueue.wait();
        // no need to use "while" here because only one barber
        current = customerQueue.removeFirst();
    }

    // hair cut the current customer

    synchronized (current) {
        current.done = true;
        // this flag helps to avoid needing nested monitor
        current.notify();
    }
}
```

03. CONSISTENCY CONDITIONS

Formalizing a Parallel System

- *Operation*: A single invocation/response pair of a single method of a single shared object by a process
  - *e* being an operation
  - proc(*e*): The invoking process
  - obj(*e*): The object
  - inv(*e*): Invocation event (*start time*)
  - resp(*e*): Reply event (*finish time*)

  wall clock time / physical time / real time
- Two invocation events are the same if invoker, invokee, parameters are the same – inv(p, read, X)
- Two response events are the same if invoker, invokee, response are the same – resp(p, read, X, 1)

- **consistency** $\rightarrow$ specifies what behaviour is allowed when a shared object is accessed by multiple processes
  - "consistent" = satisfies the specification
- **history** H $\rightarrow$ a sequence of invocations and responses ordered by wall clock time
  - for any invocation H, the corresponding response must be in H
  - each execution of a parallel system corresponds to a history and vice versa
- **sequential** $\rightarrow$ an invocation is always *immediately* followed by its response
  - no interleaving (else is **concurrent**)

  Sequential:

  $\underbrace{\text{inv(p, read, X) resp(p, read, X, 0)}}\ \underbrace{\text{inv(q, write, X, 1) resp(q, write, X, OK)}}$

  concurrent:

  inv(p, read, X) inv(q, write, X, 1) resp(p, read, X, 0) resp(q, write, X, OK)

- a history H is **legal** $\rightarrow$ if all responses satisfies the sequential semantics of the data type
  - **sequential semantics** $\rightarrow$ the semantics you would get if there is *only one process* accessing that data type
  - possible for sequential history to not be legal
    - e.g. x=0, P1 writes 1 to x, P2 reads 0 from x (if it were the same thread it would have been the same value)
- process p's **process subhistory** of H, $H|p$ $\rightarrow$ the subsequence of all events of p
  - process subhistory is always sequential
- object o's **object subhistory** of H, $H|o$ $\rightarrow$ the subsequence of all events of o
- two histories are **equivalent** $\rightarrow$ if they have the exact same set of events
  - same events $\Rightarrow$ implies all responses are the same
  - may be different ordering of events (only care about responses)
- **process/program order** $\rightarrow$ a partial order among all events
  - within the same process, process order is the same as execution order
  - no other additional orderings
- **sequential consistency** $\rightarrow$ equivalent to some legal sequential history that preserves process order
  - *(Lamport's definition)* results are same as in some sequential order & preserves program order

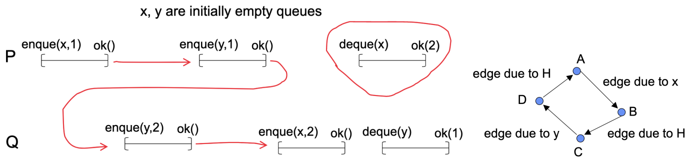
Linearisability

- stronger than sequential consistency
- **external order** $\rightarrow$ a history H induces the "<" partial order among operations
  - $o1 < o2 \iff$ the response of o1 appears in H before the invocation of o2
    - aka o1 finishes before o2 starts
  - preserves external order $\Rightarrow$ preserves program order
- **linearisability** $\rightarrow$ sequentially consistent (with some legal sequential history S) and S preserves the external order in H
  - *(alternate definition)* The execution is equivalent to some execution such that each operation happens instantaneously at some point between the invocation and response
    - for every operation in the execution, you can find a linearisation point between the invocation and response event
  - linearisable $\Rightarrow$ sequentially consistent

local property

- linearisability is a local property
  - H is linearisable $\iff$ for any object x, $H|x$ is linearisable
  - useful because you can reason about objects instead
- sequential consistency is not a local property
  - H may not be sequentially consistent, but H|x and H|y can be sequentially consistent

  Sequential Consistency is Not Local Property

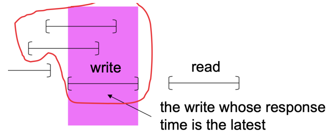


proof: linearisability is a local property

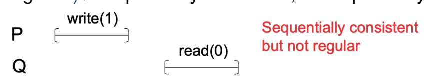
- using a directed graph: directed edge from $o1 \rightarrow o2$ if
  - $o1$ and $o2$ are on the same object x and $o1$ is before $o2$ when linearising $H|x$ ($o1 \rightarrow o2$ due to obj)
  - $o1 < o2$ in external order ($o1 \rightarrow o2$ due to H)
- any topological sorting of the graph gives us a legal sequential history S
- any cycle must be composed of
  - edges to some object x (≈ 1 edge since H|x is equivalent S with a total order)
  - edges due to some H (≈ 1 edge since partial order induced by H is transitive)
  - edges due to some object y (≈ 1 edge)
  - edges due to some H (≈ 1 edge)

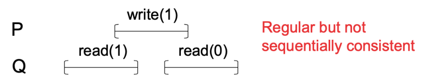
Consistency definitions for registers

- **register** $\rightarrow$ ADT: a single value that can be read and written
  - **atomic** $\rightarrow$ if the implementation always ensures linearisability of the history
  - **sequentially consistent** $\rightarrow$ if the implementation always ensures sequential consistency of the history
  - **regular** $\rightarrow$ when a read
    - not overlap with any write, the read returns the value written by one of the most recent writes
    - overlaps with one or more writes, the read returns the value written by one of the most recent writes OR the value written by one of the overlapping writes



  - **safe** $\rightarrow$ if the implementation always ensures that
    - when a read does not overlap with any write, it returns the value written by one of the most recent writes
    - when a read overlaps with one or more writes, it can return anything
- **!** atomic $\Rightarrow$ regular $\Rightarrow$ safe
- regular $\not\Rightarrow$ sequentially consistent; sequentially consistent $\not\Rightarrow$ regular



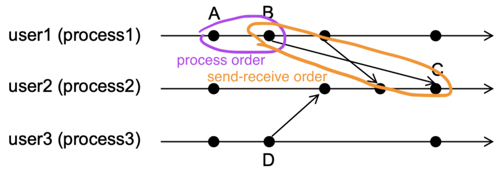


04. MODELS & CLOCKS

- process can perform 3 kinds of atomic events/actions
  - **local computation**
  - **send** a single message to a single process
  - **receive** a single message from a single process
- **communication model**
  - point-to-point (to send to multiple processes: multiple send events)
  - error-free, infinite buffer
  - potentially out of order
- **software clocks**
  - capture event ordering that are visible to users who do not have physical clocks
  - allows a protocol to infer ordering among events

visible orderings

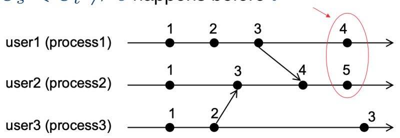
- **process order** $\rightarrow$ if A and B are on the same process, I can tell that A is before B
- **send-receive order** $\rightarrow$ the send event must be before the receive event
- **transitivity** $\rightarrow$ if A<B and B<C, then A<C



- **happened-before** relation (denoted $e \rightarrow f$) captures the ordering that is visible to users when there is no physical clock
  - partial order among events
- **concurrent-with** relation (denoted $e || f$) if $\neg(e \rightarrow f) \wedge \neg(f \rightarrow e)$

Logical Clocks

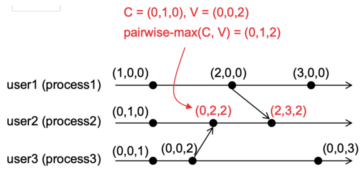
- each event has a single integer as its logical clock value
- each process has a local counter C
- protocol
  - increment C at each **local computation** and **send event**
  - **send event:** attaches the logical clock value V to the message
  - **receive event:** $C = \max(C, V) + 1$
- if event s happens before event $t \Rightarrow C_s < C_t$
  - $C_s < C_t \not\Rightarrow s$ happens before t



- for total order, extend with process number
  - $s.c$ denotes the value of c in state s, $s.p$ indicates the process it belongs to
  - the timestamp of any event is a tuple $(s.c, s.p)$
  - $(s.c, s.p) < (t.c, t.p) \iff (s.c < t.c) \vee ((s.c = t.c) \wedge (s.p < t.p))$

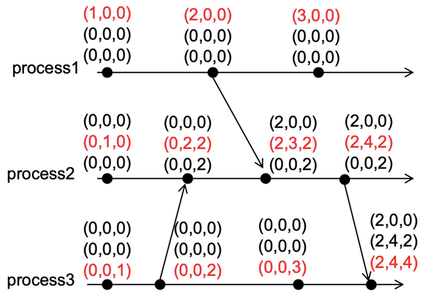
Vector Clocks

- event s happens-before event $t \iff C_s < C_t$
- each event has a vector of n integers as its vector clock value
  - $v1 = v2$ if all n fields are the same
  - $v1 \leq v2$ if every field in $v1$ is less than or equal to the corresponding field in $v2$
    - e.g. (3,1,5)$\leq$(4,1,7)
  - $v1 < v2$ if $v1 \leq v2$ and $v1 \neq v2$ (< is NOT a total order here)
- each process i has a local vector C
- protocol
  - increment C[i] at each **local computation** and **send** event
    - i^{th} entry is the principle entry
    - process i is the only process that can create new values for the i^{th} entry
  - **send event:** vector clock value V is attached to the message
  - **receive event:** $C = \text{pairwise-max}(C, V)$; C++



Matrix Clocks

- each event has 1 vector clock for each process
  - the i^{th} process on vector i is called process i's principle vector
- protocol
  - for *principal vector* C on process i,
    - increment C[i] at each **local computation** and **send** event
    - **send event:** all n vectors are attached to the message
    - **receive event:** $C = \text{pairwise-max}(C, V)$; C[i]++
      - where V is the principle vector of the sender
  - for *non-principal vector* C on process i
    - **receive event:** $C = \text{pairwise-max}(C, V)$;

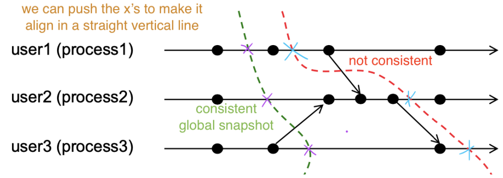


05. GLOBAL SNAPSHOT

- captures a snapshot of local states on n processes such that the global snapshot could have happened sometime in the past (user cannot tell the difference)

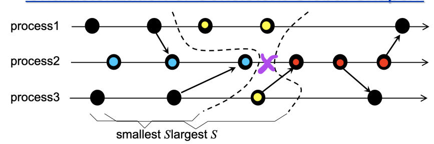
Consistent Snapshot

- **consistent snapshot** $\rightarrow$ a snapshot of local states on n processes such that the global snapshot could have happened sometime in the past
  - can have outgoing (L $\rightarrow$ R) arrows, but can't have incoming (R $\rightarrow$ L) arrows



- **global snapshot** $\rightarrow$ a set of events such that if $e2$ is in the set and $e1$ is before $e2$ in process order, then $e1$ must be in the set
  - a collection of local snapshots
- **consistent local snapshot** $\rightarrow$ a global snapshot s.t. if $e2$ is in the set and $e1$ is before $e2$ in send-receive order, then $e1$ must be in the set
  - aka global snapshot + any receive event in the set has its corresponding send event in the set
  - transitive relations implied

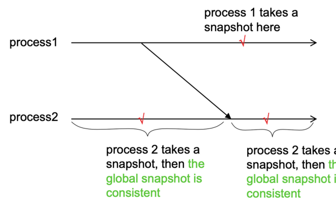
Characterization of Consistent Global Snapsht

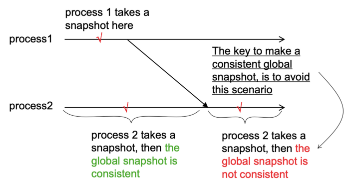


- There are other valid choices for S
- A key characterization: 3 kinds of events
  - Must be in S: Those happened before blue
  - Cannot be S: Those happened after red
  - May or may not be S: Other events

capturing a CGS

- communication model
  - no message loss
  - communication channels are unidirectional (model bidirectional channels as 2 unidirectional channels)
  - FIFO delivery on each channel
- ensuring FIFO
  - each process maintains a message number counter for each channel and stamps each message sent
  - receiver will only deliver messages in order

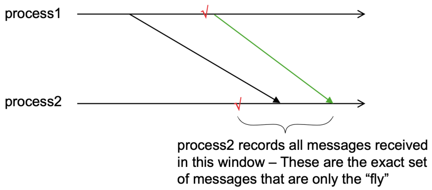




Chandy & Lamport's Protocol

- each process is either
  - red - has taken local snapshot
  - white - has not taken local snapshot
- protocol initiated by a single process by turning itself from white to red

- once a process turns red, immediately send out Marker messages to all other processes
- upon receiving Marker, process turns red
- total $n * (n-1)$ Marker messages
- requires FIFO - marker messages are sent on the same channel as actual messages
- **on-the-fly** messages: sent before sender's local snapshot, received after receiver's local snapshot



06. MESSAGE ORDERING

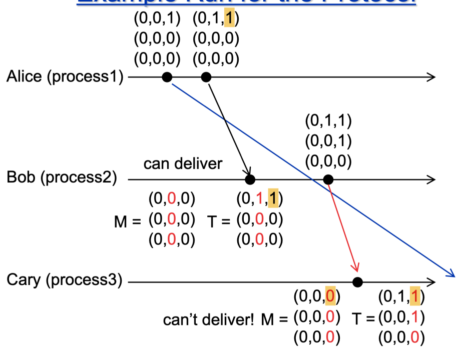
Causal Order

- **causal order** $\to$ if $s1$ happened before $s2$, and $r1$ and $r2$ are on the same process, then $r1$ must be before $r2$
  - $s_1 \to s_2 \Rightarrow \neg(r_2 \prec r_1)$
- **FIFO** $\to$ any 2 messages from process P_i to P_j are received in the same order as they are sent
  - $s_i \prec s_j \Rightarrow \neg(r_j \prec r_i)$
- $e \prec f$ denotes e occurred before f in the same process
- $s_i \rightsquigarrow r_i$ denotes s_i is the send event corresponding to receive event r_i

protocol: ensure causal order

- each process maintains a n by n matrix M
  - $M[i,j]$ = # of messages sent from i to j, as known by local (current) process
- when process i sends a message to process j,
  - on process i: $M[i,j]$++
  - piggyback M on the message
- when process j (with local matrix M) receives a message from process i with matrix T piggybacked,
  - set $M = \text{pairwise-max}(M,T)$ if $\begin{cases} T[k,j] \le M[k,j] & \text{for all } k \ne i \\ T[i,j] = M[i,j]+1 \end{cases}$
    - intuition: $M[i,j]$ on process j takes on consecutive values
    - if the entry is >1 larger than the local entry, it means that there is another message in propagation
      - we only care about column j - [i, j] should have a difference of 1
  - else, delay the message

Example Run for the Protocol



- M never decreases!
- for broadcast messages, same protocol (modelled as n point-to-point messages)

Total Ordering of Broadcast Messages

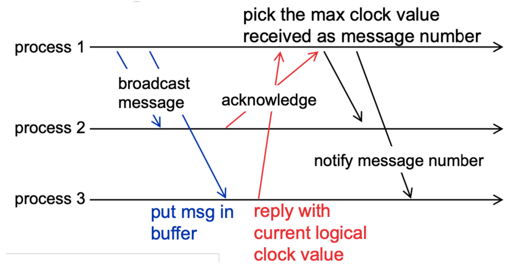
- **broadcast** $\to$ sent to all (including the sender itself)
- **total ordering** $\to$ all messages delivered to all processes in exactly the same order (aka **atomic broadcast**)
  - i.e. if every message is assigned a number, the number has to be consistent across all users
  - total ordering only applies to broadcast messages
- total ordering $\not\Rightarrow$ causal ordering
  - causal ordering $\not\Rightarrow$ total ordering

Coordinator protocol

- a special process is assigned as the **coordinator**
- to broadcast a message:
  - send a message to the coordinator
  - coordinator assigns a seq # to the message
  - coordinator forwards the message to all processes with the sequence number
    - messages delivered according to seq# order
- problem: coordinator has too much control

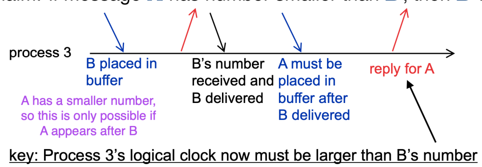
Skeen's Algorithm

- each process maintains
  - logical clock
  - message buffer for undelivered messages
- a message in the buffer is delivered/removed if
  - all messages in the buffer have been assigned numbers
  - this message has the smallest number
- protocol
  - process broadcasts a message
  - receiving processes put the message in buffer and reply (ACK) with their current logical clock value
  - sending process picks the max clock value as message number and notifies (broadcasts) message number



correctness proof

- claim: all messages will be assigned message numbers
- claim: all messages will be delivered
- claim: if message A has number smaller than B, then B is delivered after A



- Suppose A is delivered on process 3 after B.
- Then A must have been placed in buffer after B was delivered
- A must have a number larger than B – Contradiction.

07. LEADER ELECTION

- leader election trivially solves mutual exclusion and total order broadcast

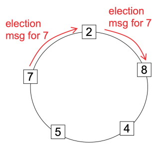
Leader Election on a Ring

anonymous ring

- **anonymous ring** $\rightarrow$ no unique identifiers
  - all must run the same algorithm (otherwise the algo itself is the unique ID)
  - node can only send messages to its neighbours
- leader election impossible using deterministic algorithms
  - same: initial state, state at each step, final state, algorithm on each node

Chang-Roberts Algorithm

- idea: largest ID is the leader
  - each node has a unique ID
  - nodes only send messages clockwise
- protocol:
  - node sends an election message with its own ID clockwise
  - a node forwards an election message if the ID is larger than its own ID
    - otherwise discard
  - a node becomes a leader if it sees its own election message



- **performance**
  - best case: $2n - 1$ messages
  - worst case: $\frac{n(n+1)}{2}$
- **average case:** $O(n \log n)$
  - taken over all possible orderings of nodes, each ordering having the same probability
    - $(n-1)!$ total orderings of the IDs
    - let $x_k =$ number of messages caused by node k's election message
    - we want to find $E[x_k]$ for all k from 1 to n
  - by linearity of expectation, $E[\sum x_k] = \sum E[x_k]$
  - $E[x_k] = \sum_{i=1}^{k}(i \cdot Pr[x_k = i])$
  - $Pr[x_k = 1] = Pr[\text{next node has larger ID than } k] = \frac{n-k}{n-1}$
  - $E[x_k] \leq E[y] = \frac{1}{p} = \frac{n-1}{n-k}$ where y is a random variable denoting the number of lottery tickets we need to buy until winning the lottery (of probability p) for the first time
  - $\sum_{k=1}^{n} E[X_k] = n + \sum_{k=1}^{n-1} E[X_k] < n + \sum_{k=1}^{n-1} \frac{n-1}{n-k} = n + (n-1)\sum_{k=1}^{n-1} \frac{1}{k} = n + (n-1)O(\log n) = O(n \log n)$

Leader Election on a General Graph

n is known

- complete graph
  - each node sends its ID to all other node
  - wait until you receive n IDs - biggest ID wins
- any connected graph
  - flood your ID to all other nodes
  - ask neighbours to recursively forward ID to other neighbours
  - wait until you receive n IDs - biggest ID wins

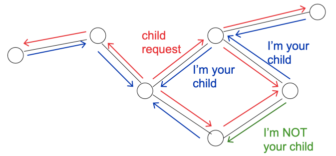
n is unknown

- complete graph: n must be known
- any connected graph: use an auxiliary protocol to calculate n
  - initiated by any node that wants to know n
  - establish a spanning tree starting from the initiator

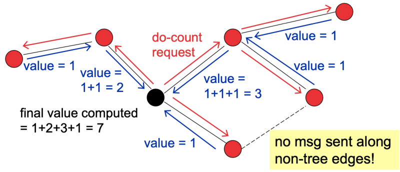
spanning tree to calculate n

- goal: each node knows its parents and children
- it's fine if multiple nodes initiate this process - you're just counting n

1. **construct spanning tree:** initiator sends child requests
   - if node has not ACKed, sends ACK and send child requests to its neighbours
   - if node has already ACKed, reject the request



2. **count nodes:** initiator sends do-count request
   - recursive: children will respond with 1 + the number of children they have



08. DISTRIBUTED CONSENSUS

goal

- **termination** $\rightarrow$ all nodes (that have not failed) eventually decide
- **agreement** $\rightarrow$ all nodes that decide should decide on the same value
  - if a node agrees then crashes, still satisfy the agreement
- **validity** $\rightarrow$ if all nodes have the same initial input, that value should be the only possible decision value
  - otherwise can decide on anything (but still satisfy Agreement)

v0. no failures

- trivial

v1. Node crash failures

model

- failure model
  - $\times$ node **crash failures**
    - node runs the algo, but stops executing at some arbitrary point in time
  - $\checkmark$ communication channels are reliable
- timing model
  - $\checkmark$ communication channels are **synchronous**
    - message delay has a *known* upper bound x
    - node processing delay has a *known* upper bound y (given as an input)

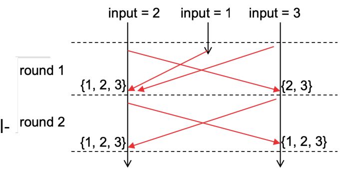
synchronous systems and rounds

- each round, every process:
  1. does some local computation (local processing delay)
  2. sends one message to every other process (message propagation delay)
  3. receives one message from every other process
- **round duration** = clock error + msg propagation delay + local processing delay
  - assume each process has a physical clock with some bounded clock error
- start a new round every ROUND_DURATION seconds
  - according to local clock
- each message has a round number attached to it
  - a message sent in a round must be received by the end of that round on the receiver
  - if receiver receives a message before it even starts the round: buffer the message until round starts

protocol

- protocol: at each round, keep forwarding the values received
  - each process sends its input to all others
  - pick the min (or max)

- $f + 1$ rounds needed for f failures
  - 1 round will be failure-free
  - lower bound $\Omega(f)$
- f must be an input to the protocol
  - user indicates maximum number of failures to be tolerated



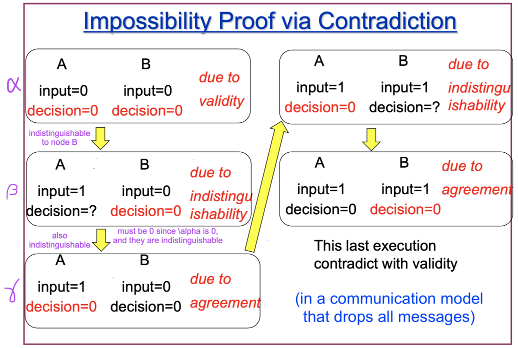
agreement proof

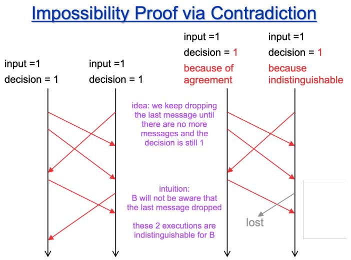
- with $f + 1$ rounds and f failures, there must be at least one good round
  - claim: At the end of any good round r, all non-faulty nodes during round r have the same S
  - claim: Suppose r is a good round. The value of S on any non-faulty nodes does not change during any round after r.
  - claim: All nonfaulty processes at round f+1 will have the same S

v2. Link failures (Coordinated Attack)

model

- failure model
  - ✓ nodes do not fail
  - × communication channels may fail - drop arbitrary (unbounded) # of msgs
- timing model
  - ✓ synchronous
- goal: termination/agreement/validity
  - impossible to achieve these using a deterministic algorithm
    - cos communication channel can drop all messages
    - execution α is **indistinguishable** from execution $\beta \rightarrow$ if the node sees the same messages and inputs in both execution





v2.1. Weakened goal

still impossible using deterministic algo

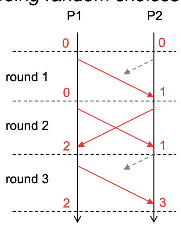
- if all nodes start with 0, the decision = 0
- if all nodes start with 1 and no message is lost during execution, decision = 1
- otherwise, any consensus

v2.2. Limited disagreement (small error)

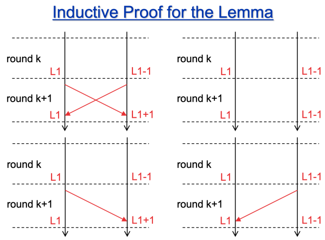
- **goal**
  - termination
  - *agreement:* all nodes decide on the same value with probability $1 - \epsilon$
  - *validity*
    - if all nodes start with 0, decision = 0
    - if all nodes start with 1 and no message is lost throughout, decision = 1
    - else: anything
- **adversary** maximises error probability
  - ✓ set inputs of the processes
  - ✓ cause message losses
  - × does not know the outcome of any randomisation

randomised algorithm

- 2 processes (P1, P2), predetermined number (r) of rounds
  - adversary determines which messages are lost *before seeing random choices*
- protocol
  - P1 picks a random integer $bar \in [1..r]$
  - each P1, P2 maintains a $level$ variable L1, L2
    - L1 and L2 differ by at most 1
  - each round: send each other messages
    - attach input, bar and $level$ to each message
    - P1: set L1 = L2+1
      - (symmetric) same for P2
  - **!** L1/L2 never decreases!



- decision rule: after r rounds, P1/P2 decide on 1 $\iff$
  - it knows its input *and* the other process input are both 1
  - it knows bar (always true for P1)
  - its $level \geq bar$
- error probability $\epsilon = \frac{1}{r}$



When does error occur?

- For P1 and P2 to decide on different values: One must decide on 1 while the other decide on 0
  - For someone to decide on 1, P1's input and P2's input must be both 1
- Case 1:
  - P1 sees P2's input, but P2 does not see P1's input or does not see bar
  - Then L1 = 1 and L2 = 0. Error occurs only when bar = 1.
- Case 2:
  - P2 sees P1's input and bar, but P1 does not see P2's input
  - Then L1 = 0 and L2 = 1. Error occurs only when bar = 1.
- Case 3:
  - P1 sees P2's input, and P2 sees P1's input and bar
  - Define Lmax = max(L1, L2)
  - Error occurs only when bar = Lmax.

v3. Node crash failures + Asynchronous

model

- failure model
  - ✓ reliable channel
  - × node crash failures
- timing model
  - **asynchronous** $\rightarrow$ process delay and message delay are *finite but unbounded*
  - can no longer define a round
- goal: termination/agreement/validity
- **FLP theorem** (Fischer, Lynch, Paterson) $\rightarrow$ the distributed consensus problem under asynchronous communication model is impossible to solve even with a single node crash failure
  - fundamentally, because the protocol cannot accurately detect node failure

formalisms of FLP theorem

- global state of a system = all process states + message system state
  - message system captures on-the-fly messages: $\{(p, m) | \text{message } m \text{ on the fly to process } p\}$
  - all messages are distinct
  - sending and receiving operations are adding and removing content from the message system, and changing process local state
- each step given a global state is fully described by p's receiving m
  - (p, m) is an event
    - events are inputs to the state machine, that cause state transitions
  - event e can be applied to the global state G if either $m == null$ or (p, m) is in the message system
- execution of any protocol is an *infinite sequence of events*
  - process fail = *finite* number of steps
- schedule σ is a sequence of events that captures the execution of a protocol
  - $G' = \sigma(G)$ means we apply σ to G to get G'
  - must make sure σ can be applied to G (aka G' is **reachable** from G if $\exists \sigma$ such that $G' = \sigma(G)$)
- messages have unbounded but finite delay - every message is eventually delivered

v4. Node Byzantine failures

model

- failure model
  - ✓ reliable channels
  - × **byzantine failures** $\rightarrow$ nodes can behave unexpectedly and arbitrarily
    - must consider the worst case where nodes intentionally break the protocol
- timing model
  - ✓ synchronous
- goal: termination/agreement/validity for *non-faulty nodes* only
  - *agreement:* bad nodes can't be forced to decide
  - *validity:* if all the good nodes have the same initial input, that value should be the only possible decision value
- **byzantine consensus threshold** for n processes, f possible byzantine failures
  - if $n \leq 3f$, then the byzantine consensus problem cannot be solved

protocol for $n \geq 4f + 1$

- intuition
  - *rotating coordinator:* process i is the coordinator for phase i
  - coordinator sends a proposal to all processes
  - a phase is a **deciding phase** if the coordinator is nonfaulty
    - agreement after a phase with a good coordinator
    - if the coordinator is non faulty, all processes see the proposal
- protocol
  - $f + 1$ phases (cos at most f bad nodes)
    - each phase is 2 rounds
  - set local value = input
  - each phase
    - round 1: all-to-all broadcast
      - send your local value to all processes
    - round 2: coordinator round
      - set proposal to majority ($> n/2$)
      - send the proposal to everyone
    - decide whether to listen to the coordinator
      - if overwhelming majority ($> \frac{n}{2} + f$), set local value to majority
      - else, set local value = coordinator's proposal

proof

- Lemma 1: if all good processes have the same local value at the beginning of a given phase, then this remains true at the end of a phase
  - since we have $n - f$ good processes and $n - f > \frac{n}{2} + f$, they will be the overwhelming majority
- Lemma 2: if the coordinator in a phase is good, agreement will be achieved at the end of that phase
  - case 1: coordinator sees (and proposes) a majority=x
    - then x appears (>n/2) times, of which >n/2-f must be from good processes
    - thus other processes will see x (>n/2-f) times
    - so no other value can be overwhelming majority
  - case 2: coordinator receives equal distribution
    - then it's impossible for anyone to see an overwhelming majority
    - for coordinator, no value x appears (>n/2) times
    - thus for other processes, impossible for any $y \neq x$ to appear (>n/2+f) times
- *termination:* after f+1 phases
- *validity:* follows from lemma 1
- *agreement:*
  - with f+1 phases, at least one is a deciding phase
  - by Lemma 2, all good processes will agree at deciding phase
  - by Lemma 1, after a deciding phase, additional phases will not disrupt the agreement achieved

Summary

Failure Model and Timing Model	Consensus Protocol
Ver 0: No node or link failures	Trivial – all-to-all broadcast
Ver 1: Node crash failures; Channels are reliable; Synchronous;	(f+1)-round protocol can tolerate f crash failures
Ver 2: No node failures; Channels may drop messages (the coordinated attack problem)	Impossible without error Randomized algorithm with 1/r error prob
Ver 3: Node crash failures; Channels are reliable; Asynchronous;	Impossible (the FLP theorem)
Ver 4: Node Byzantine failures; Channels are reliable; Synchronous; (the Byzantine Generals problem)	If n ≤ 3f, impossible. If n ≥ 4f + 1, we have a (2f+2)-round protocol. How about 3f+1 ≤ n ≤ 4f ?

10. SELF-STABILISATION

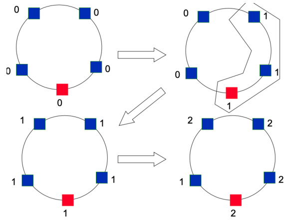
- state of a distributed system is either legal or illegal
  - based on application semantics
- **self-stabilising** $\rightarrow$ if
  - starting from any (legal or illegal) state, the protocol will eventually reach a legal state if there are no more faults
  - once in a legal state, it will only transit to a legal state unless there are faults
- typically runs in the background and never stops

Rotating Privilege Problem

- a ring of n processes
  - each process can only communicate with neighbours
- at any time, only one node may have the privilege
- privilege is like a token

algorithm

- each process i has local integer variable V_i where $0 \leq V_i \leq k, \quad k \geq n$
- one red process and some blue processes (allocate before running the algorithm)



- red process
  - retrieve value L of clockwise neighbour
  - if $V == L$: (has privilege)
    - increment V++ with mod k
- blue process
  - retrieve value L of clockwise neighbour
  - if $V \neq L$: (has privilege)
    - set $V = L$

legal states

- Lemma: there are only 2 legal states
  1. all n values same $\Rightarrow$ red process privilege
  2. only 2 different values forming 2 consecutive bands, with one band starting from the red process $\Rightarrow$ blue process privilege

Self-Stabilising Spanning Tree

- given n processes connected by an undirected graph and one special process P1, construct a spanning tree rooted at P1
- each process maintains 2 variables: **parent** and **dist** (to root, ≥ 0)
  - faults: wrong value of these variables

algorithm

- P1 repeatedly executes 'dist=0; parent=null;'
- all other nodes periodically execute
  - retrieve dist from all neighbours
    - set own $dist = 1 + \min($neighbour dists$)$
  - set own parent = neighbour with smallest dist
    - break tie: based on ID (e.g. smaller ID)

proof

- **phase** $\rightarrow$ minimum time period where each process has executed its code at least once (can be more than once)
- assume that the topology doesn't change
  - i.e. no additional faults - we only need to reason about self-stabilising algos if there are no additional faults
- let
  - A_i be the **level** of process i (length of the shortest path from i to root)
    - A_i will not change
  - $dist_i$ be the value of dist on i
    - $dist_i$ may change
- properties of levels of the nodes
  1. a node at level X has at least one neighbour in level $X - 1$
     - if there is a neighbour with level $< X - 1$, then the node can have a smaller level $< X$
  2. a node at level X can only have neighbours in level $X - 1, X, X + 1$
     - for X-1: same proof as (1)
     - if there is a neighbour with >X+1, then that node can be X+1 by going through the node instead
- lemma: at the end of phase r,
  1. any process i whose $A_i \leq r - 1$, has $dist_i = A_i$
     - if level is < r, the distance value must become correct (i.e. dist=level)
  2. any process i whose $A_i \geq r$, has $dist_i \geq r$
     - for the remaining processes, the distance value is $\geq r$ and may still be incorrect
- **proof:** by induction
  - base: holds for $r = 1$
  - assume lemma holds at phase r and consider phase $r + 1$

common self-stabilisation proof technique

- Step 1: Prove that the t actions will not roll back what is already achieved so far by phase r (no backward move)
- Step 2: Prove that at some point, each node will achieve more (forward move)
- Step 3: Prove that the t actions will not roll back the effects of the forward move after the forward move happens (no backward move after the forward move)