Mid-Term Course Work CM1015 / Computational Mathematics

UNIVERSITY OF LONDON

CM1015

COMPUTER SCIENCE

Computational Mathematics

2022-2023

INSTRUCTIONS TO STUDENTS:

This paper consists of 5 questions. You should answer **ALL** the questions.

There are 100 marks available on this paper. The marks for each question are indicated at the end of the part in [.] brackets. **Full marks will be awarded for complete answers to a total of 5 questions**.

All answers need to be written clearly

The point of this assessment is to give you the opportunity to consolidate your learning and to assess your understanding of the topics. You do need to submit your answers as a pdf document (probably a single document is best), or photos of your work, or your work properly formatted using the maths mode of your word processor).

The total work is worth 100 marks distributed as follows:

* 15 marks for topic-1 (Number Bases)

* 30 marks for topic-2 (Sequences, Series and Mathematical induction)

* 15 marks for topic-3 (Modular Arithmetic)

* 20 marks for topic-4 (Angles, Triangles and Trigonometry)

* 20 marks for topic-5 (Graph Sketching and Kinematics)

Question 1

(a) Find the largest decimal number that can be written in base 2 using 6 binary digits. Show your work. [2]

(b) What is the value of $\frac{1}{32}$ in binary? Show your work. [2]

(c) Perform the base calculation for the following number system:

i. Perform the base 16 calculation $7B \times 5E$, writing your answer in base 16. Show your work. [1]

ii. Perform the base 16 calculation $1CA - 90$, writing your answer in base 16. Show your work. [1]

(d) The multiplication of two positive integer numbers is 9222 and the difference between them is 19. Find the sum of the numbers. Show your answer in details. [4]

(e) Evaluate $\sqrt[3]{512_8}$ as a number in the decimal system. Show your work. [1]

(f) Find the answer for the following questions: Show your work. [4]

i. $(11011)_2 + (11011)_2$

ii. $(10111)_2 - (10001)_2$

iii. $(11011)_2 \times (11011)_2$

iv. $(11011)_2 \div (110)_2$

Question 2

(a) A profit of a seller grows to $\$11.23$ in his first week. Each week, his profit raise by 6% more than what he made in the week before. Use your knowledge about sequences and series topic to find by how much does he raise profit in eight weeks, including the first week? Show your work. [4]

(b) A local employer uses his bike to go to work each day. In the first minute, he travels 0.15 km. After that, in each following minute, he travels 75% of the distance travelled during the previous minute. If the distance from his accommodation to the destination is 0.7 km. If he leaves at 8:00 am, would he be at work by 8:15am? Use your Knowledge about sequences and series to prove your work. [4]

(c) Use mathematical induction to prove that for all $n \in N$,

$$\frac{d^n}{dx^n} sinx = sin(x + n\frac{\pi}{2}), \quad (x \in \mathbf{R})$$

[4]

(d) Does the sequence whose n^{th} term is

$$an = \frac{n+1}{n-1}^n$$

converge? If so, find $\lim_{n\to\infty} a_n$. Show your work. [4]

(e) Find four numbers forming a geometric sequence such that the second term is 35 less than the first term and the third term is 560 more than the fourth term. Show your work. [5]

(f) If a sequence has a first term $a_1 = 12$ and a common difference $d = -7$, write the formula that describes this sequence. Use the formula of the arithmetic sequence. Show your work. [4]

(g) Find the sum of the first five terms of the geometric sequence in which $a_1 = 3$ and $r = -2$. Show your work. [5]

Question 3

(a) Find the following multiplicative inverses, if any. Show your work. [3]

i. 2 (mod 13)

ii. 238 (mod 7)

iii. 538 (mod 17)

(b) If $a \equiv b(mod\, k)$, that is if a is congruent to b in modulo k, the congruence relation formally written $a = kn + b$
Show that if $a \equiv c(mod\, n), b \equiv d(mod\, n)$, then: [6]

i. $a + b \equiv (c + d)(mod\, n)$

ii. $a - b \equiv (c - d)(mod\, n)$

iii. $a \times b \equiv (c \times d)(mod\, n)$

(c) What is the time 100 hours after 7 a.m.? Show your work. [3]

(d) Find the remainder when 2^{81} is divided by 17. Show your work. [3]

Question 4

(a) Given a triangle ABC with sides a = 48 cm, b = 48 cm and c =54 cm find the angles of A, B and C. Show your work. [4]

(b) Sketch the graph of the given function. What is the period of the function? Show your work. [2]

i. $y = cos\frac{\pi x}{2}$

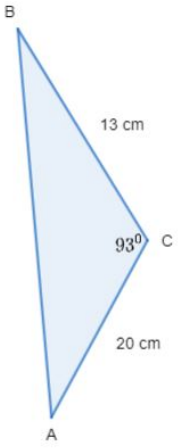
ii. $y = sin\frac{x}{2}$

(c) Convert the following degree measures into radian: [2]

i. $510°$

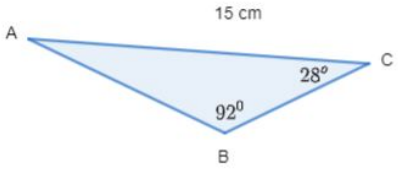
ii. $-240°$

(d) Use the following figure to find the length of AB. (figure not drawn to scale). [4]



Show your work. [4]

(e) Use the following figure to find the length of AB. (figure not drawn to scale), show your work. [4]



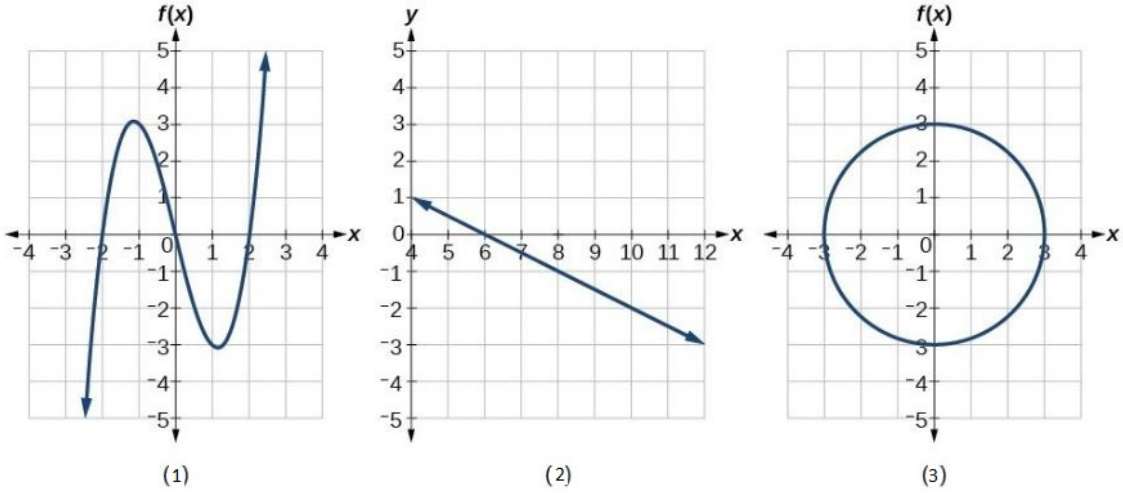
Question 5

(a) Show whether the following functions are one-to-one and onto.
$f_j : \mathbb{N} \to \mathbb{N}$: Explain your answer. [4]

i.
$$f_1(n) = n + 3$$

ii.
$$f_2(n) = \begin{cases} n-1 & \text{if n is odd} \\ n+1 & \text{if n is even} \end{cases}$$

(b) Which of the following graphs represent a function $y = f(x)$? [3]



(1) (2) (3)

(c) A Go-kart sits at rest to start a race. When the racing car driver starts, he accelerates at $2\,m/s^2$ for $5\ s$. What is the car's final velocity? Show your work. [4]

(d) Find the domain and the range for the following functions: [4]

i. $f(x) = 2^x$
ii. $y = 2 - \sqrt{-3x + 2}$
iii. $f(x) = x^2 - 1$
iv. $f(x) = (2, 10), (3, 10), (4, 20), (5, 30), (6, 40)$

(e) a ball is dropped off from a tower 19.6 meters high. How long will it take to reach the ground? Suppose that its initial velocity is zero, solve for the time it takes to reach the floor. [5]

END OF PAPER

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