UNIVERSITY OF LONDON

CM1015

COMPUTER SCIENCE

Computational Mathematics

2021-2022

INSTRUCTIONS TO STUDENTS:

This paper consists of 5 questions. You should answer **ALL** the questions.

There are 100 marks available on this paper. The marks for each question are indicated at the end of the part in [.] brackets. Full marks will be awarded for complete answers to a total of 5 questions.

All answers need to be written clearly

The point of this assessment is to give you the opportunity to consolidate your learning and to assess your understanding of the topics. You do need to submit your answers as a pdf document (probably a single document is best), or photos of your work, or your work properly formatted using the maths mode of your word processor).

The total work is worth 100 marks distributed as follows:

* 15 marks for topic-1 (Number Bases)

* 30 marks for topic-2 (Sequences, Series and Mathematical induction)

* 15 marks for topic-3 (Modular Arithmetic)

* 20 marks for topic-4 (Angles, Triangles and Trigonometry)

* 20 marks for topic-5 (Graph Sketching and Kinematics)

Question 1

(a) Here are three binary numbers:
1011110 , 1110101 and 1010011
Working with binary, [4]

    i. Add up all three numbers

    ii. Add up the two larger numbers

    iii. Add up the two smaller numbers

    iv. Take the smallest number away from the largest number.

(b) Find the value of x in binary for each of the following: [2]

    i. $110 + 1101 = \frac{x}{101}$

    ii. $100x = 11011$

(c) For each of the following numbers, list all number bases that could be used to write the number. Consider the bases (2, 5, 8, 10, 13 and 16) only. [4]

    i. A101

    ii. 4104

    iii. 777777

    iv. 8910F

(d) Convert the following numbers using number system conversions. Show your working: [5]

    i. 11101_2 to base 10

    ii. $AB.C_{16}$ to base 8

    iii. 11.0011_2 to base 8

   iv. 11.11_8 to base 2

    v. 26655_8 to base 16

Question 2

(a) The first term of a geometric sequence is 4, while 2 is the first term of an arithmetic sequence. If the common ratio of the geometric sequence equals the common difference of the arithmetic sequence and their third terms are equal, find the common ratio, common difference and the first four terms of both sequences. [6]

(b) A geometric progression is given by $2, -1, \frac{1}{2}, -\frac{1}{4} \ldots$.
Find the general formula in terms of n. [4]

(c) The 1st and 10th terms of an arithmetic series are -1 and 10, respectively. Find the sum of the first 10 terms. [4]

(d) A geometric sequence, x_n, is given by $x_n = -3(2)^{n-1}$.
Show that the sum of the first n terms, S_n, is given by $S_n = 3(1 - 2^n)$. [3]

(e) Using mathematical induction, prove that $n + 4 < n + 9$ for all values of $n \in N$. [4]

(f) Given the sequence with general term $a_n = \frac{4n-5}{2n}$, find [6]

    i. a_5

    ii. a_{-2}

    iii. a_{100}

(g) b_n is a geometric sequence with $b_5 = \frac{1}{32}$ and $b_8 = 4^{-4}$. Find the common ratio. [3]
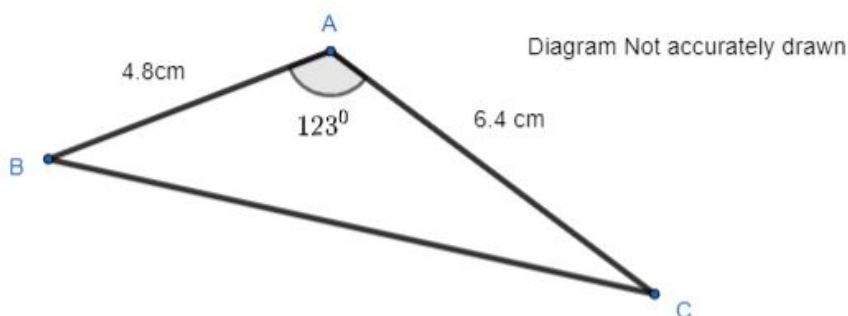
Question 3

   (a) What is the least positive value of x such that $89 \equiv (x+3)\ (mod\ 4)$?   [3]

   (b) If x is congruent to 11 modulo 19 then, $7x-3$ is congruent to which number modulo 19?   [4]

   (c) Evaluate $-97\ \mod 11$.   [2]

   (d) Find the modulo 4 residue of 311.   [3]

   (e) What is the the remainder of 17×18 when it is divided by 19. Show your working.   [3]

Question 4

(a) Given a triangle with angles A, B and C and sides with length a, b and c. Angle $C = 30°$, length of side $c = 5.5$ cm and length of side $a = 4$ cm, find the value of angle A.   [5]

(b) A triangle has sides $a = 2$ and $b = 3$ and angle $C = 40°$. Find side c and the sine of angle B. Show your working.   [5]
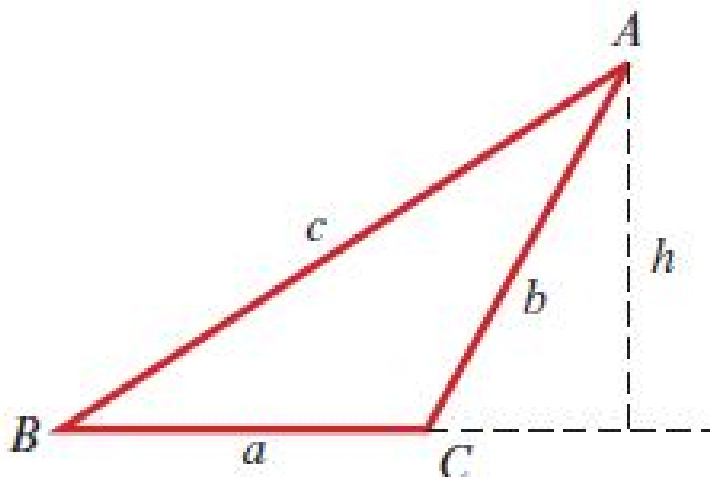
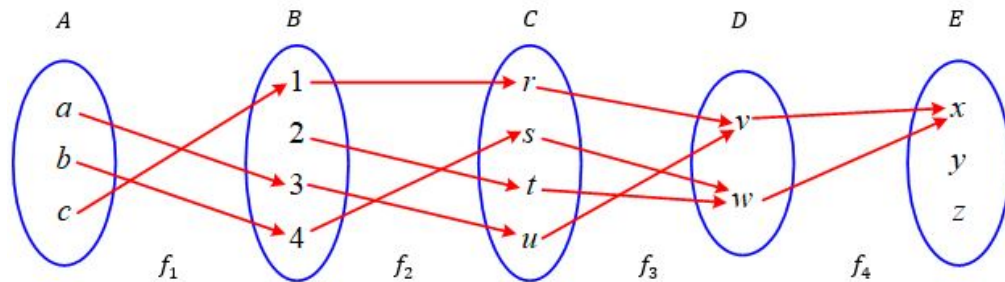(c) Use the cosine rule to determine the side opposite to the known angle A.   [5]

Diagram Not accurately drawn

4.8cm

123^0

6.4 cm

(d) In triangle ABC, side $c = 42$ cm, side $a = 37$ cm and side $b = 26$ cm. Calculate the value of angle A.   [5]

Question 5

(a) The following figure defines four functions:
$f_1 : A \rightarrow B,\ f_2 : B \rightarrow C, f_3 : C \rightarrow D$ and $f_4 : D \rightarrow E$ [8]

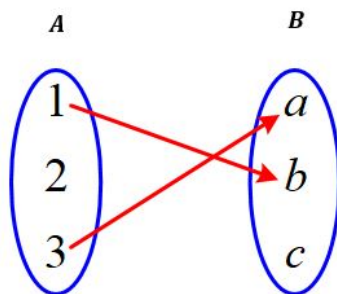
Which of the functions are:

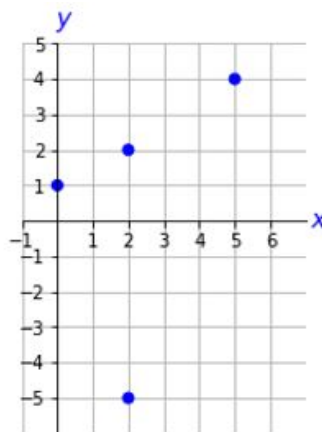
  i. injective

  ii. surjective
     explain your answer to get a full mark.

(b) State whether or not each of the following four figures i, ii, iii and iv defines
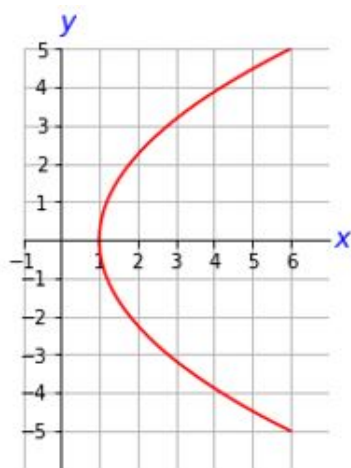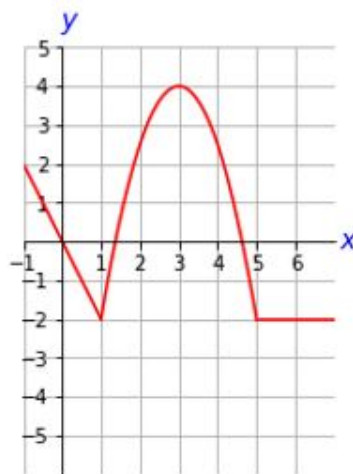a function. Explain your answer.   [4]
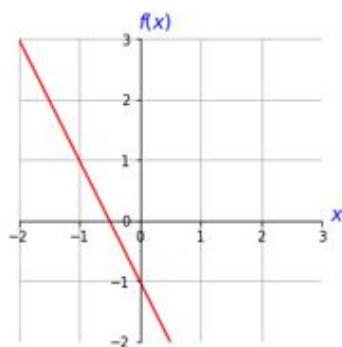
(iii)

(iv)

(c) Consider $f(x)$ and $g(x)$ which are represented by the following graph and table, respectively: [4]

x	$g(x)$
−1	3
0	1
1	−1
2	−3
3	−5

 i. Do they represent the same function? Explain your answer.

 ii. What is the expression (formula) of each one?

(d) The acceleration due to gravity at the surface of Moon is 1.67 m/s^2. If a stone is thrown upward with a speed of 10 m/s, find [4]

 i. the time required to reach the highest point

 ii. the maximum height reached by the stone

 iii. the speed of the stone at $t = 8\ s$

 iv. the displacement of the stone at $t = 8\ s$.

END OF PAPER

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