Mid-Term Course Work CM1015 / Computational Mathematics

UNIVERSITY OF LONDON

CM1015

COMPUTER SCIENCE

Computational Mathematics

2020-2021

INSTRUCTIONS TO STUDENTS:

This paper consists of 5 questions. You should answer **ALL** the questions.

There are 100 marks available on this paper. The marks for each question are indicated at the end of the part in [.] brackets. Full marks will be awarded for complete answers to a total of 5 questions.

All answers need to be written clearly

The point of this assessment is to give you the opportunity to consolidate your learning and to assess your understanding of the topics. You do need to submit your answers as a pdf document (probably a single document is best), or photos of your work, or your work properly formatted using the maths mode of your word processor).

The total work is worth 100 marks distributed as follows:

* 15 marks for topic-1 (Number Bases)

* 30 marks for topic-2 (Sequences, Series and Mathematical induction)

* 15 marks for topic-3 (Modular Arithmetic)

* 20 marks for topic-4 (Angles, Triangles and Trigonometry)

* 20 marks for topic-5 (Graph Sketching and Kinematics)

Question 1

(a) A given number in base x can be converted to any other base y. According to the expansion method, if **abc.de** is any given number in base x, then write its value in base 10. [3]

(b) Convert the following numbers using number system conversions, show your answer in details: [6]

i. $(723)_8$ to hexa decimal system

ii. $(0.ABDF)_{16}$to decimal system

iii. Convert 0.375 to binary system

iv. Which digits from (0,1,2,3,4,5) are not allowed in Quinary system (base 5) representation.

v. $(11010.1011)_2$ to hexadecimal.

vi. $(257)_{10}$ to the binary system.

(c) Consider the binary number 10.0011 [4]

i. Convert the above number to the decimal system

ii. What are the place values of the digits 1 in the number 0.0011_2

iii. what is the sum of (1+1+1+1) in binary system

iv. calculate 101 divided by 10 using long division.

(d) Which one is the correct representation of a binary number from the following? [2]

i. 1101

ii. $(214)_2$

iii. $(0000)_2$

iv. $(11)^2$

Question 2

(a) Is $a_n = \frac{3n+2}{n-4}$ a general term of a sequence? Why? [2]

(b) Which term of the sequence with general term $\frac{3n-1}{5n+7}$ is $\frac{7}{12}$? [2]

(c) An arithmetic sequence has its 4^{th} term equal to 18 and its 12^{th} term equal to 50. Find its 99^{th} term. [4]

(d) State whether the following sequences are arithmetic, geometric or not any of them. Find the common ratio if it is a geometric sequence and find the common difference d if it is an arithmetic sequence. Then, find the next two terms. [6]

i. $-3, 3, -3, 3$

ii. $b_n = n^2 + 3$

iii. $\frac{-1}{2}, \frac{-5}{6}, \frac{-7}{6}$

(e) Consider the geometric sequence (b_n) with $b_1 = \frac{1}{9}$ and $q = 3$. Is 243 a term of this sequence? [3]

(f) The nineteenth term of a sequence is -52, and the fourth term is -7. The difference between consecutive terms in the sequence is constant. Find the 201^{st} term. [3]

(g) Show whether the following sequence is convergent or divergent.
$lim_{n\to\infty}(\frac{n-1}{n})$ [2]

(h) Is the following numbers 1,-4,9,-16,... represent a sequence, if so, find a formula for the n^{th} term of the sequence.

[2]

(i) Show by mathematical induction that for all positive integers n,
$\frac{1}{2} + \frac{1}{2^2} + ... + \frac{1}{2^n} = 1 - \frac{1}{2^n}$ [4]

(j) Find the remainder when 3^{123} is divided by 7. [2]

Question 3

(a) State whether the following statements are false or true, explain your answer: [6]

i. Given any integers a, b, c and any positive integer n
If $a \equiv b\,(mod\,n)$ and $b \equiv c(mod\,n)$, then $a \equiv c(mod\,n)$.

ii. Suppose $a \equiv b(mod\,n)$ and $c \equiv d(mod\,n)$,then
$a + c \equiv b + c(mod\,n)$.

iii. $7x \equiv 12(mod\,7)$.

(b) Find the least positive value of x such that:

$71 \equiv x\,(mod\,8)$ [3]

(c) Calculate the multiplicative inverse of 168 in modulo 83. [3]

(d) Calculate the inverse of 4 modulo 15. Show your steps. [3]

Question 4

(a) A triangle has sides a = 2 and b = 3 and angle C = 60.
Find [4]

i. the length of side c.

ii. Find the sine of angle B using sine rules.

(b) If we have a triangle which has one of its side c = 2 and angles A = $\pi/4$ and B = $\pi/3$. Workout the length a of the side opposite A. [4]

(c) XYZ is a right angled triangle with $Y = 90°$. Given that $y = 85$, $\sin X = \frac{77}{85}$, find z, $Cos(Z)$ and the angle of Z. [6]

(d) Let g be a function with its domain $(0, \infty)$, defined by $g(x) = \frac{1}{x}$. [6]

i. Sketch the graph of g.

ii. Is g continuous at other points of its domain?

Question 5

(a) Consider the function $f : Z \to Z$ given by

$$f(x) = \begin{cases} n+1, & \text{if } n \text{ is even} \\ n-3, & \text{if } n \text{ is odd} \end{cases}$$

[6]

i. Is f injective? Prove your answer

ii. Is f surjective? Prove your answer

(b) The function $f : R \to R$ is defined as follows:

$$f(x) = \begin{cases} -x, & \text{if } x < 0 \\ x^2, & \text{if } 0 \leq x \leq 1. \\ 1, & \text{if } x > 1 \end{cases}$$

Plot the function, and say whether it is Bijective function or not. Explain your answer. [5]

(c) The velocity of a particle moving along the x axis varies according to the expression $v_x = 40 - 5t^2$, where v_x is in meters per second and t is in seconds.
Find the average acceleration in the time interval $t = 0$ to $t = 2$ sec. [4]

(d) An object moving with uniform acceleration has a velocity of $12\,cm/s$ in the positive x direction when its x coordinate is $3.00\,cm$. If its x coordinate 2 seconds later is $-5\,cm$, what is its acceleration? [5]

END OF PAPER

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