UNIVERSITY OF LONDON

GOLDSMITHS COLLEGE

Department of Computing

B. Sc. Examination 2018

IS51026B
Numerical Maths

Duration: 2 hours 15 minutes

Date and time:

This paper is in two parts: part A and part B. You should answer ALL questions from part A and TWO questions from part B. Part A carries 40 marks, and each question from part B carries 30 marks. The marks for each part of a question are indicated at the end of the part in [.] brackets.

There are 100 marks available on this paper.

Electronic calculators must not be programmed prior to the examination. Calculators which display graphics, text or algebraic equations are not allowed.

THIS PAPER MUST NOT BE REMOVED FROM THE EXAMINATION ROOM

Part A

Multiple choice

Question 1 Each question has one correct answer

(a) What is the decimal representation of 321_8?

i. 83_{10}
ii. 418_{10}
iii. 209_{10}
iv. none of the above

[2]

(b) What is the fractional representation of the recurring decimal in simplest form 4.239239...?

i. $\frac{4235}{999}$
ii. $\frac{239}{999}$
iii. $\frac{847}{200}$
iv. none of the above

[2]

(c) What is the multiplicative inverse of 5 in modulo 7?

i. 1
ii. 2
iii. 3
iv. 4

[2]

(d) A right angled triangle ABC has sides $a = 5$ cm, $b = 9$ cm and c is the hypotenuse. The size of angle A in radians is

i. 0.507
ii. 1.064
iii. 10.3 cm
iv. This triangle does not exist

[2]

(e) A triangle XYZ has sides $x = 8$ cm, $y = 7$ cm and angle $Y = 1.13$ radians. The size of angle X is:

i. 0.441
ii. 1.111
iii. 7.88 cm
iv. This triangle does not exist

[2]

(f) Convert 1.7 radians to degrees

i. 97.4^o
ii. 48.7^o
iii. 194.8^o
iv. 33.7^o

[2]

(g) The frequency of $f(x) = 2\cos(\pi + x)$ is

i. 2
ii. 2π
iii. $\frac{1}{2}$
iv. $\frac{1}{2\pi}$

[2]

(h) The amplitude of $f(x) = 2\cos(\pi + x)$ is

i. $\frac{1}{2}$
ii. $\frac{1}{2\pi}$
iii. 2π
iv. 2

[2]

(i) $\log_2 6 + \log_2 \frac{1}{2}$ is equal to:

i. 6.5
ii. $\log_2 6.5$
iii. $\log_2 3$
iv. 3

[2]

(j) $\log_9 3$ is equal to

i. $\frac{1}{\log_3 9}$
ii. $-\log_3 9$
iii. $\frac{1}{3}$
iv. is not defined

[2]

(k) The graph of $\log_2 x$:

i. has a x-intercept of 1
ii. has a y-intercept of 0
iii. passes through the point $(1, 2)$
iv. passes through the point $(0, 0)$

[2]

(l) Calculate the following limit: $\lim_{x\to\infty} \frac{x^5+x^3-7}{2x^5-3x+1}$.

i. -7
ii. ∞
iii. $\frac{1}{2}$
iv. is not defined

[2]

(m) Given $y = x^2(x^2 + x)$

i. $\frac{dy}{dx}$=$x^4 + x^3$
ii. $\frac{dy}{dx}$=$2x(2x + 1)$
iii. $\frac{dy}{dx}$=$4x^3 + 3x^2$
iv. $\frac{dy}{dx}$ is not defined

[2]

(n) Given $y = \frac{x^2+x}{x^2}$

i. $\frac{dy}{dx} = 1 + \frac{1}{x}$
ii. $\frac{dy}{dx} = -\frac{1}{x^2}$
iii. $\frac{dy}{dx} = \frac{2x+1}{2x}$
iv. $\frac{dy}{dx}$ is not defined

[2]

(o) Convert the vector $\underline{u} = \begin{pmatrix} 2 \\ 5 \end{pmatrix}$ in cartesian coordinates to polar coordinates

i. $(4.58, 1.19)$

ii. $(5.39, 1.19)$

iii. $\sqrt{21}$

iv. $\sqrt{29}$

[2]

(p) You are given vectors $\underline{u} = \begin{pmatrix} 5 \\ 0 \\ 2 \end{pmatrix}$ and $\underline{v} = \begin{pmatrix} 2 \\ 5 \\ -1 \end{pmatrix}$

$\underline{u} - \underline{v}$ is equal to

i. $\begin{pmatrix} 7 \\ 5 \\ 1 \end{pmatrix}$

ii. $\begin{pmatrix} 3 \\ -5 \\ 3 \end{pmatrix}$

iii. $\begin{pmatrix} 3 \\ 5 \\ 3 \end{pmatrix}$

iv. $\begin{pmatrix} 10 \\ 0 \\ -2 \end{pmatrix}$

[2]

(q) Find M^{-1}, the inverse of M where $M = \begin{pmatrix} 1 & 0 & 3 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{pmatrix}$

i. $\begin{pmatrix} 1 & 0 & -1 \\ 0 & 1 & 3 \\ 0 & 0 & 1 \end{pmatrix}$

ii. $\begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & -3 \\ 0 & 0 & 1 \end{pmatrix}$

iii. is undefined

iv. none of the above

[2]

(r) The following matrix represents which of the following transformations? $\begin{pmatrix} 2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

i. A translation
ii. A rotation
iii. A reflection
iv. A scaling

[2]

(s) Given complex numbers $z_1 = 2 + i$ and $z_2 = i$ find $z_1 \times z_2$.

i. $1 + 2i$
ii. $-1 + 2i$
iii. $1 - 2i$
iv. $-1 - 2i$

[2]

(t) Given complex numbers $z_1 = 2 + i$ and $z_2 = i$ find $\frac{z_1}{z_2}$.

i. $\frac{1+2i}{3}$
ii. $\frac{1+2i}{5}$
iii. $1 - 2i$
iv. $-1 + 2i$

[2]

Part B

Question 2 Bases, Modular Arithmetic & Trigonometry

(a) i. Express the decimal number $(177)_{10}$ in base 8 [1]

ii. Express the decimal number $(11.125)_{10}$ as a binary number [2]

iii. Express the hexadecimal number $(32.8)_{16}$ as a decimal number [2]

iv. Express the octal number $(262.24)_8$ as

(1) a binary number

(2) a hexadecimal number [3]

v. Working in base 8 and showing all your working, compute the following:

$$(4763)_8 + (332)_8 - (4606)_8$$

[2]

(b) i. Find the smallest positive integer modulo 17 that is congruent to

(1) 271

(2)1277 [2]

ii. Find the remainder on division by 17 of

(1) $271 - 1277$

(2) 271×1277

(3) 271^{35} [6]

iii. Find the following

(1) the additive inverse of 15 modulo 17

(2) the multiplicative inverse of 15 modulo 17 [2]

(c) i. Triangle ABC has side $a = 16cm$, side $b = 10cm$ and angle $C = 1.65$ radians Find

(1) the length of side c

(2) the size of angle A

(3) the size of angle B [4]

ii. Given $f(x) = \sin(3x + \frac{\pi}{2})$ and $g(x) = 3\cos x$

(1) Plot the graphs of $f(x)$ and $g(x)$ for $-\pi \leq x \leq \pi$ [4]

(2) By using your graph or otherwise, find all the values of x for $-\pi \leq x \leq \pi$ for which $\sin(3x + \frac{\pi}{2}) = 3\cos x$ [2]

Question 3 Functions, Graph Sketching & Vectors

(a) i. Find numerical values for the following

(1) $\log_{10} 100$

(2) $\log_{10} 0.001$

(3) $\log_{1000} 10$ [3]

ii. Give the functions $f(x) = 2^x - 1$ and $g(x) = 1 + log_2 x$

(1) Plot the graphs of $f(x)$ and $g(x)$ [4]

(2) Find the inverse functions $f^{-1}(x)$ and $g^{-1}(x)$ [3]

(b) i. Find the following limits

(1) $\lim_{x\to 2} \frac{x^2-1}{x^3-x}$

(2) $\lim_{x\to 0^-} \frac{x^2-1}{x^3-x}$

(3) $\lim_{x\to 0^+} \frac{x^2-1}{x^3-x}$

(4) $\lim_{x\to \infty} \frac{x^2-1}{x^3-x}$ [4]

ii. Given the function $f(x) = (x-1)(x^2+x+1)$

(1) Find the value or values of x for which $f(x) = 0$
(note $(x^2+x+1) \geq 0$ for all x)

(2) Differentiate $f(x)$

(3) Hence find any stationary points of $f(x)$ and determine their nature

(4) Sketch $f(x)$ [6]

(c) Given $\underline{v}_1 = \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix}$ and $\underline{v}_2 = \begin{pmatrix} -1 \\ 0 \\ 2 \end{pmatrix}$

i. Rewrite $\underline{v}_1$ and $\underline{v}_2$ in terms of standard unit vectors

ii. Find the magnitudes of $\underline{v}_1$ and $\underline{v}_2$

iii. Find the dot product of $\underline{v}_1$ and $\underline{v}_2$

iv. Hence find the angle between $\underline{v}_1$ and $\underline{v}_2$

v. Find $\underline{v}_3$ the cross product (vector product) of $\underline{v}_1$ and $\underline{v}_2$

[10]

Question 4 Matrices & Complex Numbers

(a) Let A be a 3x3 homogeneous transformation matrix corresponding to a scaling of the x and y-coordinates by a factor of 2 and a factor of 3 respectively, let B be a 3x3 homogeneous tranformation matrix corresponding to a translation of the x and y coordinates by 1 and -1 respectively and let C be a 3x3 homogeneous transformation matrix corresponding to a clockwise rotation about the z-axis through an angle $\frac{\pi}{6}$

i. Find matrices A, B and C [3]

ii. How would the transformation represented by the matrix B transform the following three points which represent a triangle in the Cartesian space: (1,0), (2,0) and (2,1)? [3]

iii. Find the inverse matrices A^{-1} and C^{-1} [2]

iv. Find the single matrix D which represents the transformation represented by matrix C followed by the transformation represented by matrix B [3]

v. Find the inverse of the homogeneous transformation matrix $E = \begin{pmatrix} -1 & 0 & 2 \\ 0 & -1 & 3 \\ 0 & 0 & 1 \end{pmatrix}$ [4]

(b) Given complex numbers $z_1 = 3 - i$ and $z_2 = 2 + 3i$

i. represent z_1 and z_2 on an Argand diagram [1]

ii. Find

(1) $z_1 + z_2$

(2) $z_1 - z_2$

(3) $z_1 \times z_2$

(4) $\overline{z_2}$

(5) $\frac{z_1}{z_2}$ [5]

iii. Convert z_1

(1) to polar form

(2) to exponential form [3]

iv. Hence find ${z_1}^3$ [2]

v. Given $z_3 = -1$

(1) Find all the roots ${z_3}^{\frac{1}{3}}$ [3]

(2) Represent all the roots ${z_3}^{\frac{1}{3}}$ on an Argand diagram [1]