UNIVERSITY OF LONDON

GOLDSMITHS COLLEGE

Department of Computing

B. Sc. Examination 2017

IS51026A/IS51026B
Numerical Maths

Duration: 2 hours 15 minutes

Date and time:

This paper is in two parts: part A and part B. You should answer ALL questions from part A and TWO questions from part B. Part A carries 40 marks, and each question from part B carries 30 marks. The marks for each part of a question are indicated at the end of the part in [.] brackets.

There are 100 marks available on this paper.

THIS PAPER MUST NOT BE REMOVED FROM THE EXAMINATION ROOM

Part A
Multiple choice

Question 1 This question has one correct answer

(a) What is the decimal value of binary sequence 11111111_2?

i. 255

ii. 127

iii. 511

iv. none of the above

[2]

(b) What is the fractional representation of the following recurring decimal 1.405405?

i. $\frac{1405}{1000}$

ii. $\frac{281}{200}$

iii. $\frac{1405}{999}$

iv. $\frac{52}{37}$

[2]

(c) What is the smallest positive number that is congruent to 8095×471 in modulo 256?

i. 3,812,745

ii. 14,893

iii. 137

iv. 32

[2]

(d) A triangle XYZ has sides $x = 6$, $y = 8$ and angle $X = 42^o$. The size of angle Y is:

i. 30^o

ii. 63^o

iii. 49^o

iv. 37^o

[2]

(e) Convert 9^o to radians.

i. $\frac{\pi}{2}$
ii. $\frac{\pi}{20}$
iii. $\frac{\pi}{4}$
iv. $\frac{\pi}{10}$

[2]

(f) Convert (5, 0) to polar coordinates.

i. $(5, 0)$
ii. $(5, \pi)$
iii. $(-5, 0)$
iv. none of the above

[2]

(g) $\log_2 (2^6)$ is equal to:

i. 12
ii. 2^6
iii. 8
iv. 6

[2]

(h) The graph of $f(x) = 2^x$:

i. has y-intercept of 0
ii. has x-intercept of 1
iii. passes through the point $(0, 1)$
iv. passes through the point $(1, 0)$

[2]

(i) Given $y = x^5 + 4x^3 - 2x^2$:

i. $\frac{dy}{dx}$ $=5x + 12x - 4x$
ii. $\frac{dy}{dx}$ $=5x^4 + 12x^2 - 4x$
iii. $\frac{dy}{dx}$ $=13x$
iv. $\frac{dy}{dx}$ $=x^4 + 4x^2 - 2x^1$

[2]

(j) You may use the following kinematics equations (suvat equations)

$s = ut + \frac{1}{2}at^2$

$s = \frac{1}{2}(u + v)t$

$v^2 = u^2 + 2as$

$v = u + at$

A particle moves with constant acceleration. It's final velocity is $8ms^{-1}$ and its acceleration is $-2ms^{-2}$. Find the initial velocity if the particle travels a distance of 8 metres.

i. $9.8ms^{-1}$

ii. $5.7ms^{-1}$

iii. $6.9ms^{-1}$

iv. $8.9ms^{-1}$

[2]

(k) Calculate the following limit: $\lim_{x\to 5} \frac{x-5}{x^2-25}$.

i. 10

ii. does not exist

iii. 0.1

iv. 0

[2]

(l) Given $y = \sin 5x$:

i. $\frac{dy}{dx} = 5\sin 5x$

ii. $\frac{dy}{dx} = 5\cos 4x$

iii. $\frac{dy}{dx} = \cos 5x$

iv. $\frac{dy}{dx} = 5\cos 5x$

[2]

(m) Rewrite the following vector in terms of standard unit vectors: $\begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix}$

i. $2\vec{i}$-$\vec{j}$+$\vec{k}$

ii. $\begin{pmatrix} 2\vec{i} \\ -1\vec{j} \\ 1\vec{k} \end{pmatrix}$

iii. $2 - 1 + 1$

iv. none of the above

[2]

(n) Given 2 non-zero vectors $\underline{u}$ and $\underline{v}$ if $|\underline{u} \times \underline{v}| = |\underline{u}| \times |\underline{v}|$

Which of the following must be true?

i. $\underline{u}$ and $\underline{v}$ are parallel

ii. $\underline{u} = \underline{v}$

iii. $\underline{u}$ and $\underline{v}$ are perpendicular

iv. none of the above

[2]

(o) Find M^{-1}, the inverse of M where $M = \begin{pmatrix} 2 & -1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

i. $\begin{pmatrix} 0 & 1 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

ii. $\begin{pmatrix} 0 & 0 & 0 \\ 1 & 2 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

iii. $\begin{pmatrix} 0 & -1 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

iv. does not exist

[2]

(p) Given W= $\begin{pmatrix} 2 & 0 & 1 \\ 0 & 2 & -1 \\ 0 & 0 & 1 \end{pmatrix}$

Which of the following is the inverse of W?

i. $\begin{pmatrix} 1 & 0 & 2 \\ -1 & 2 & 0 \\ 1 & 0 & 0 \end{pmatrix}$

ii. $\begin{pmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 1 & -1 & 1 \end{pmatrix}$

iii. $\begin{pmatrix} \frac{1}{2} & 0 & -1 \\ 0 & \frac{1}{2} & 1 \\ 0 & 0 & 1 \end{pmatrix}$

iv. $\begin{pmatrix} \frac{1}{2} & 0 & -\frac{1}{2} \\ 0 & \frac{1}{2} & \frac{1}{2} \\ 0 & 0 & 1 \end{pmatrix}$

[2]

(q) The following matrix represents which of the following transformations? $\begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 1 \end{pmatrix}$

i. A translation

ii. A rotation

iii. A reflection

iv. A scaling

[2]

(r) Given complex numbers $z_1 = 3 + 2i$ and $z_2 = -2 - i$ find $z_1 - z_2$.

i. $5 + 3i$

ii. $-5 - 3i$

iii. $1 + i$

iv. $-1 - i$

[2]

(s) Given the complex number $z = -2 - i$ find $\overline{z}$ the complex conjugate of z.

i. $-2 + i$
ii. $2 - i$
iii. $-1 + 2i$
iv. $1 - 2i$

[2]

(t) Given the complex number $z = \sqrt{2}(\cos \pi/6 + i \sin \pi/6)$ find z^2.

i. $2(\cos \pi^2/6 + i \sin \pi^2/6)$
ii. $2(\cos \pi/3 + i \sin \pi/3)$
iii. $2\sqrt{2}(2 \cos \pi/6 + 2i \sin \pi/6)$
iv. $\sqrt{2}(\cos \pi/3 + i \sin \pi/3)$

[2]

Part B

Question 2 Bases, Modular Arithmetic & Trigonometry

(a) i. Express the decimal number $(347)_{10}$ in base 2 [1]

ii. Express the binary number $(1000111.011)_2$ as a decimal number [2]

iii. Express the decimal number $(281.75)_{10}$ as

(1) a binary number

(2) a hexadecimal number [2]

iv. Express the octal number $(574.2)_8$ as a decimal number [2]

v. Working in base 16 and showing all your working, compute the following:

$$(AB2)_{16} + (161)_{16} - (FF)_{16}$$

[3]

(b) i. Find the smallest positive integer modulo 13 that is congruent to

(1) 54

(2)271 [2]

ii. Find the remainder on division by 13 of

(1) $54 + 271$

(2) 54×271

(3) 271^{19} [6]

iii. Find the following

(1) the additive inverse of 5 modulo 13

(2) the multiplicative inverse of 5 modulo 13 [2]

(c) i. Triangle ABC is an isosceles triangle (has 2 equal sides). Side $a = 6cm$ and angle $A = 80^o$

(1) Find all 3 possible values for angle B

(2) Hence find all 3 possible values for the length of side b [2]

ii. Let $f(x) = 3\cos(x)$ and $g(x) = \sin(2x)$

(1) Find the amplitude, frequency and period for

- $f(x)$
- $g(x)$

[6]

(2) By plotting the graphs of $f(x)$ and $g(x)$, or otherwise find all the values of x between $-\pi$ and π for which $3\cos(x) - \sin(2x) = 0$ [2]

Question 3 Functions, Graph Sketching & Vectors

(a) i. Find numerical values for the following

(1) $\log_2 1024$

(2) $\log_{1024} 2$

(3) $\log_2(\frac{1}{2})$ [3]

ii. Sketch the graphs of

(1) $f(x) = 2^x$

(2) $g(x) = 2^{x-1}$ [3]

iii. Find the inverse functions

(1) $f^{-1}(x)$

(2) $g^{-1}(x)$ [4]

(b) i. Find the following limits

(1)$\lim_{x\to 0} \frac{x-4}{x^2-16}$

(2) $\lim_{x\to +4} \frac{x-4}{x^2-16}$

(3)$\lim_{x\to \infty} \frac{x-4}{x^2-16}$

(4)$\lim_{x\to -4} \frac{x-4}{x^2-16}$ [4]

ii. Given the following function $f(x) = x^3 - 3x^2$

(1) Find the values of x for which $f(x) = 0$

(2) Differentiate $f(x)$

(3) Hence find any stationary points of $f(x)$ and determine their nature

(4) Sketch $f(x)$ [6]

(c) Given $\underline{v}_1 = \begin{pmatrix} 2 \\ 3 \\ 0 \end{pmatrix}$ and $\underline{v}_2 = \begin{pmatrix} -1 \\ 0 \\ 2 \end{pmatrix}$

i. Find the magnitudes of $\underline{v}_1$ and $\underline{v}_2$

ii. Find the dot product of $\underline{v}_1$ and $\underline{v}_2$

iii. Hence find the angle between $\underline{v}_1$ and $\underline{v}_2$

iv. Find $\underline{v}_3$ and $\underline{v}_2$ the cross product (vector product) of $\underline{v}_1$ and $\underline{v}_2$

v. State the angle between $\underline{v}_3$ and $\underline{v}_1$

[10]

Question 4 Matrices & Complex Numbers

(a) Let A be a 3x3 matrix corresponding to a translation of 3 units in the x direction and -1 unit in the y direction. Let B be a 3x3 matrix corresponding to a scaling of factor 2 in the x direction and factor 3 in the y direction

i. Write down A and B [2]

ii. Find the inverse matrices A^{-1} and B^{-1} [3]

iii. Find the single matrix C which represents the transformation represented by matrix B followed by transformation represented by matrix A [3]

iv. How would the combined transformation represented by the matrix C transform the following three points which represent a triangle in the Cartesian space: (0,0), (2,0) and (2,1)? [3]

v. Find the inverse matrix C^{-1} [4]

(b) Given complex numbers $z_1 = 3 + 2i$ and $z_2 = 5 - 2i$

i. Find

(1) $z_1 + z_2$

(2) $z_1 - z_2$

(3) $z_1 \times z_2$

(4) $\frac{z_1}{z_2}$ [6]

ii. Convert z_1

(1) to polar form

(2) to exponential form [4]

iii. Hence find

(1) ${z_1}^3$

(2) All solutions to ${z_1}^{\frac{1}{3}}$ [5]